Probabilistic Nonlinear Finite Element Analysis of Reinforced Concrete Beams without Shear Reinforcement

MSc graduation Thesis

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TU Delft

Civil Engineering
Geotechnical Engineering
Petroleum Engineering

DIANA FEA

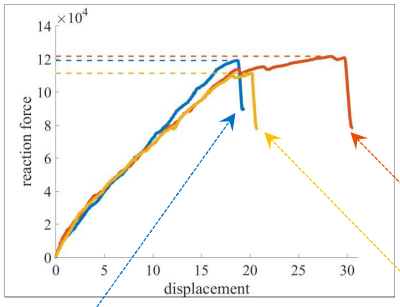
2016, November 24

Problem definition

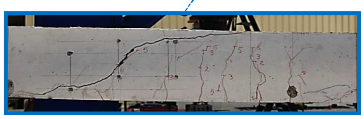
Experimental results

- similar experiments
- different failure mode

⇨ material uncertainty



beam A – flexural failure



beam B – shear failure

beam C – shear failure

Research objectives

- Tackle the effect of material uncertainty
- Explain the bi-modal experimental behavior.
- Estimate reliability measures.

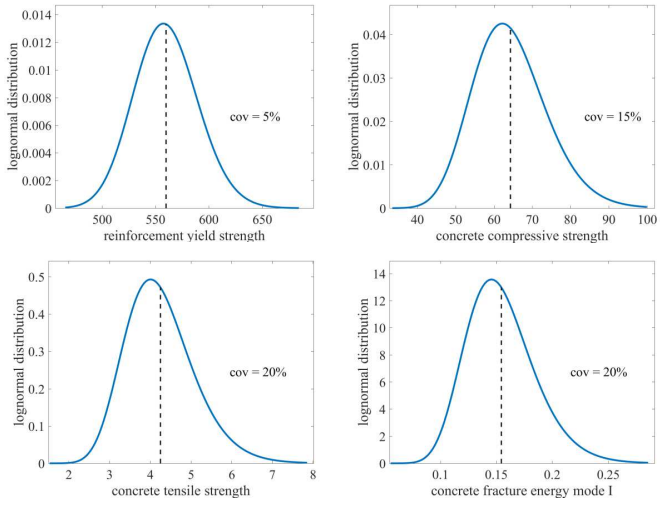
Solution strategy

- stochastic approach
- structural analysis
- quantify material uncertainty

Quantifying the material uncertainty

- random variables
- stochastic properties

fib Model Code 2010

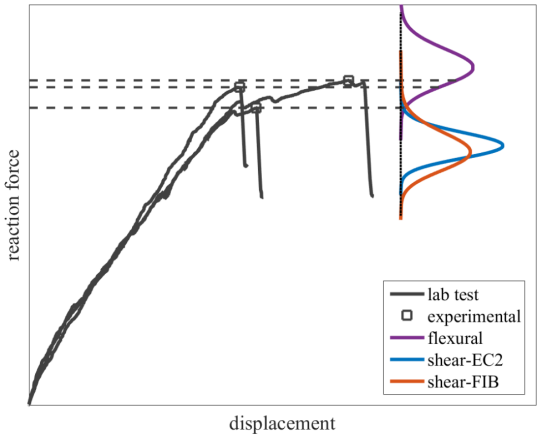


Solution strategy

- structural analysis
  - analytical models — EC2, Model Code
  - finite element modeling
- stochastic approach

Analytical models

Analytical models vs Experimental data



- Monte Carlo Simulation
- analytical models
  - shear capacity
  - flexural capacity

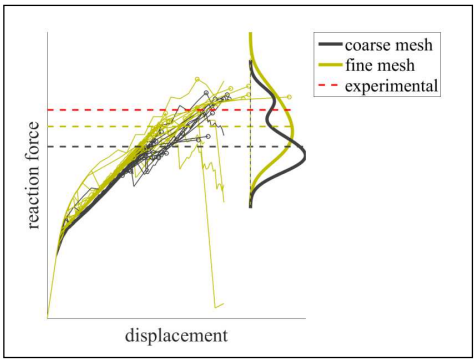
⇩

- higher prob. of occurrence: shear failure
- failure mode transition

Solution strategy

- structural analysis
  - analytical models ~~EC2~~ ~~Model Code~~
  - finite element modeling calibration
- stochastic approach

Calibrating the FE model – numerical scheme



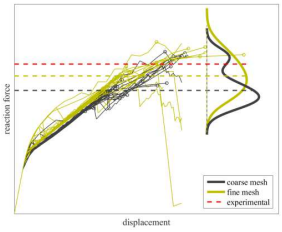
Numerical scheme	Mesh		Load		Convergence		Iterative scheme		
	aspect ratio	element size	type	step size	norm	tol.	itera-tions	stiff-ness	conver. scheme
final scheme	1	25	displa.	2.5%	energy	$1e-03$	400	secant	line-search

- mean material properties
- convergence
- stability
- vs experimental results
  - failure mode
  - accuracy

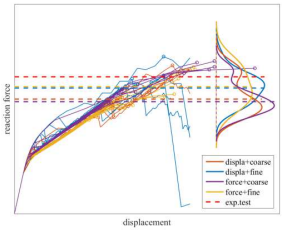
Scheme	Mesh		Load		Convergence		Iterative scheme		
	aspect ratio	element size	type	step size	norm	tol.	itera-tions	stiff-ness	conver. scheme
scheme0	1	25	displa.	5%	displa	$1e-02$	400	secant	line-search
						$1e-03$			
		50	force	1%	energy	$1e-04$		Newton-Raphson	arc-length
						$1e-03$			
					force	$1e-02$			
						$1e-03$			
					force & displa.	$1e-02$			
						$1e-03$			

Calibrating the FE model – numerical scheme

mesh



control



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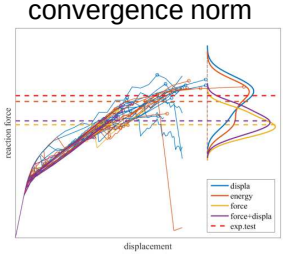
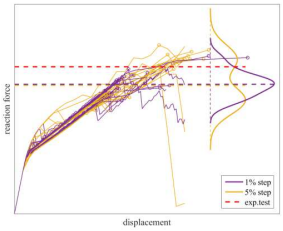
					force	$1e-02$			
						$1e-03$			
					force & displa.	$1e-02$			
						$1e-03$			

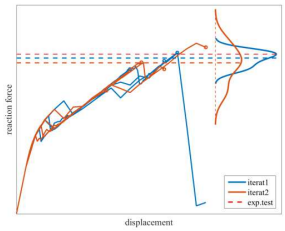
convergence norm



load increments size



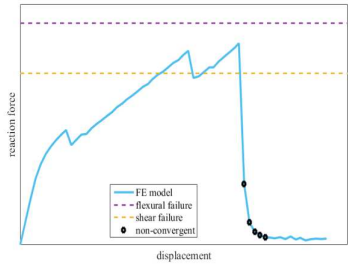
iterative method

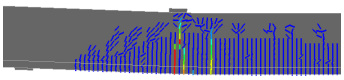


Calibrating the FE model – constitutive scheme

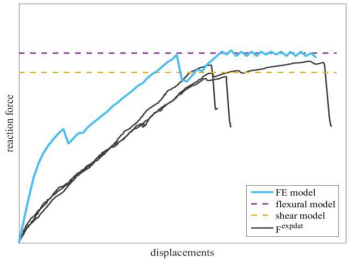
Constitutive behavior	Concrete							Steel			
	tension soften.	compress. soften.	total crack strain	shear retention	reduction Poisson	lateral confine.	lateral cracking.	σ - ε	concrete - reinforcement	concrete-steel	σ - ε
FE model	hordijk	parabolic	fixed	damage based	damage based	Selby & Vecchio	Vecchio & Collins	von Mises	embedded	interface	linear

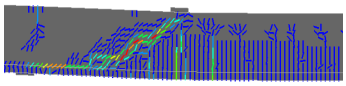
high fy



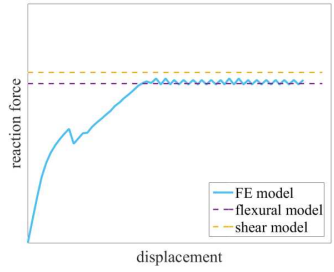


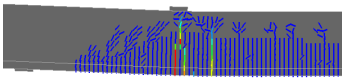
mean fy





low fy





Solution strategy

- structural analysis
  - analytical models ~~EC2~~ ~~Model Code~~
  - finite element modeling calibration
- stochastic approach
  - semi-probabilistic: ~~partial safety factor~~ ~~global safety factor~~
  - full probabilistic ~~Monte Carlo~~ Response Surface Method

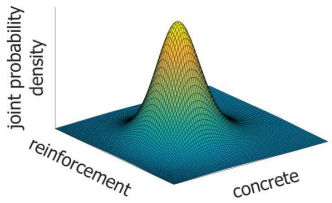
PNLFEA

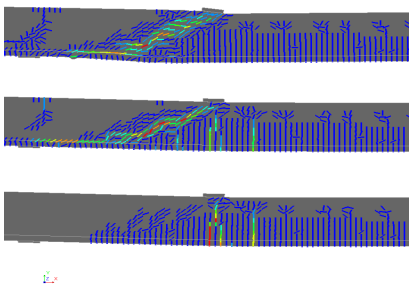
DIANA 10.0 – PROBAB

Directional Adaptive Response Surface method (DARS)

Input for PNLFEA

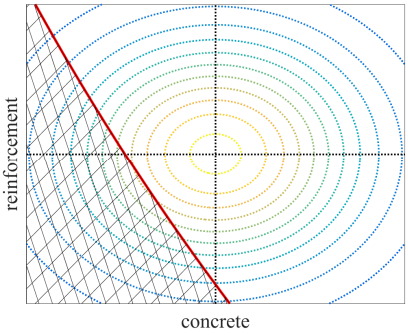
- random variables, (**X**)
  - concrete: ρ=1.00
- finite element model (calibrated)
- method
  - Directional Adaptive Response Surface (**DARS**)
- analysis parameters
  - number of samples, tolerances, etc
- limit state function

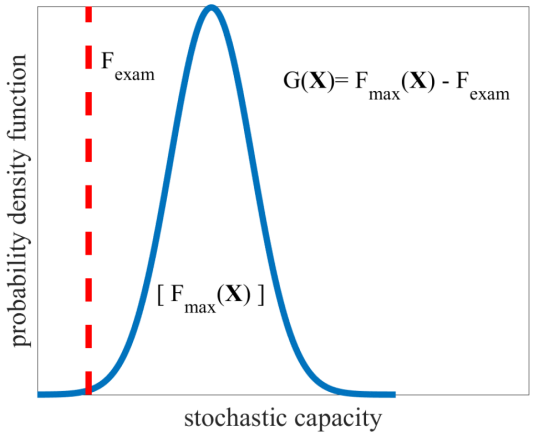




Limit state function

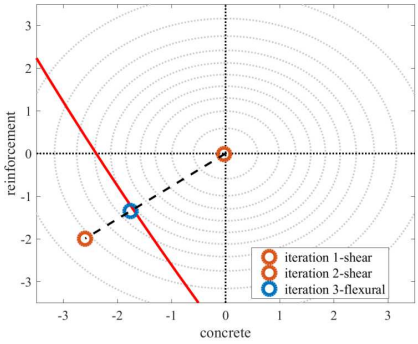
- reaction force capacity $G(\boldsymbol{X}) = F_{\max}(\boldsymbol{X}) - F_{exam}$
- limit state function, G(**X**)
  - G(**X**)>0, safe
  - G(**X**)≤0, failure

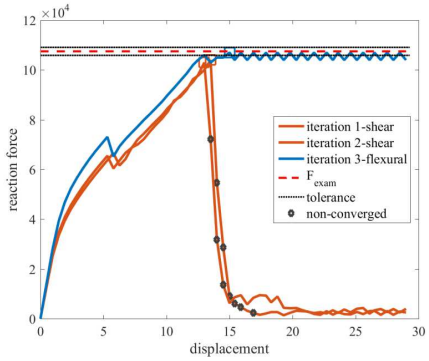




PNLFEA - DARS

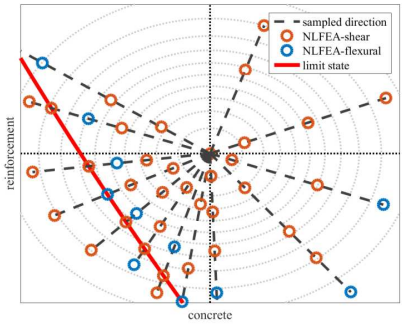
- directional sampling & line search
- NLFEA



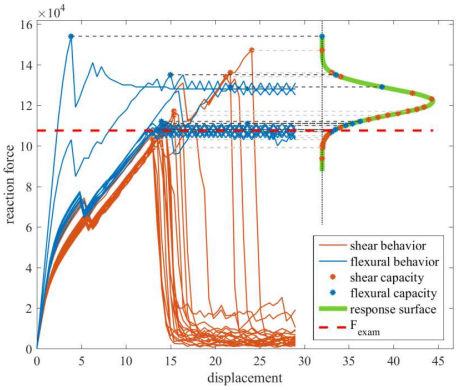


PNLFEA - DARS

- directional sampling & line search



- NLFEA



- fitting the response surface function

$$G^*(\boldsymbol{U}) = a + \sum_{i=1}^{n} b_i U_i + \sum_{i=1}^{n} c_i U_i^2 \quad = a + b_1 U_c + b_2 U_y + c_1 U_c^2 + c_2 U_y^2$$

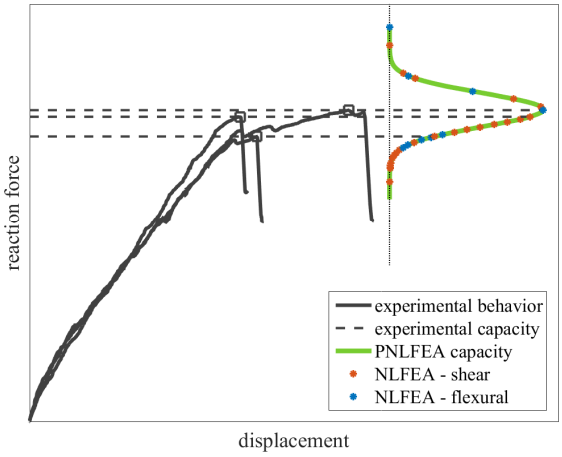
PNLFEA vs Experimental resutls

- PNLFEA (RS)

⇨ full structural behavior

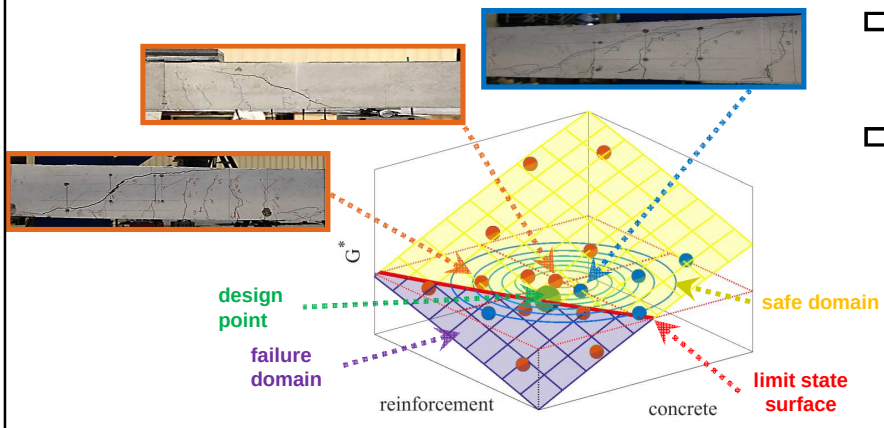
- PNLFEA vs Experimental results
  - capacity
  - failure mode

⇨ bi-modal experimental behavior



Response surface & Reliability

- Monte Carlo on Response Surface Function



Response Surface of
Reaction Force Capacity

$$\beta = 2.22$$
$$P_f = 0.0131$$

$$P_{oc.shear} = 76.34\%$$
$$P_{oc.\text{flexural}} = 23.66\%$$

⇨ actual material properties

⇨ design point

Sensitivity measures

- correlation between concrete properties: (ρ=0.85)
- constructed response surface function:

$$G^*(\mathbf{U}) = 0.135 - 0.838\cdot10^{-5}\cdot U_{f_c} + 0.562\cdot10^{-2}\cdot U_{f_{ct}} - 0.329\cdot10^{-4}\cdot U_{G_f^I}$$
$$-0.586\cdot10^{-5}\cdot U_{E_c} - 0.386\cdot10^{-2}\cdot U_{f_y}$$
$$-0.134\cdot10^{-4}\cdot U_{f_c}^2 - 0.692\cdot10^{-4}\cdot U_{f_{ct}}^2 - 0.133\cdot10^{-4}\cdot U_{G_f^I}^2$$
$$-0.132\cdot10^{-4}\cdot U_{E_c}^2 + 0.257\cdot10^{-4}\cdot U_{f_y}^2$$

- largest influence
  - concrete tensile strength
  - reinforcement yield strength

Conclusions

- Explain the experimentally observed behavior of reinforced concrete beams without shear reinforcement
  - actual material properties of experimental members
- Tackle the effect of material uncertainty
  - structural behavior for all realizations of material properties
  - sensitivity measures
- Estimate reliability measures
  - structural reliability
  - probability of occurrence of each failure mode
  - design point

Current improvements

- Response surface function
  - cross-terms
  - multiple limit states
  - multiple design-points
  - higher efficiency/robustness

$$G = a + \sum_{i=1}^{n} b_i u_i + \sum_{i=1}^{n} c_i u_i^2 + \sum_{i=1}^{n} \sum_{j=1,\, j \neq i}^{n} d_{ij} u_i u_j$$

- Model uncertainty factor
  - $\boldsymbol{\theta}$
  - FEM uncertainty
  - higher robustness

$$G(\boldsymbol{X}, \boldsymbol{\theta}) = \boldsymbol{\theta}\, F_{\max}(\boldsymbol{X}) - F_{exam}$$

Application

- Wide range of application
  - all FEM applications
  - random variables: material, geometrical, loading
  - multiple limit states: reaction force, stress, strain, displacement, etc
  - examination + design purposes
- Reinforced concrete bridges designed with old structural codes
- Advantages
  - full probabilistic: all scenarios considered
  - finite element analysis: advanced analysis, system effects

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