Computational Methodologies for the Non-linear Analysis of Concrete and Masonry Structures

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Overview

- Introduction
  - earthquake engineering problems
  - issues in finite element analysis
- Objectives
- Material models
  - combined damage/plasticity model
  - time dependent damage model
- Analysis methods
  - dynamic analysis
  - static analysis
- Applications and conclusions

Earthquake-resistant design of concrete and masonry structures

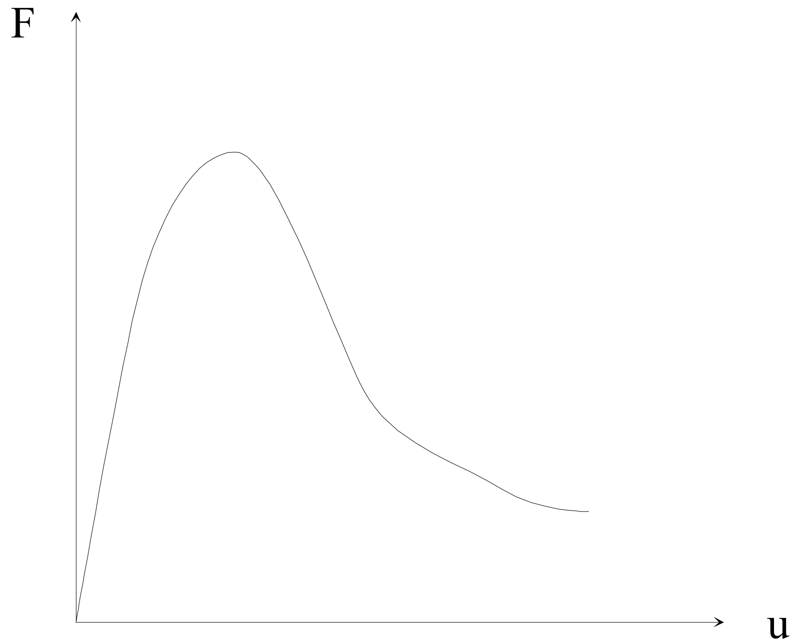
- Project motivation:
  - contribution to safer building practice in seismic regions
  - increased understanding of failure mechanisms in concrete structures and masonry structures using advanced numerical techniques

Examples of failure in masonry buildings


Examples of failure in infilled r.c. structures

Non-linear finite element structural analysis

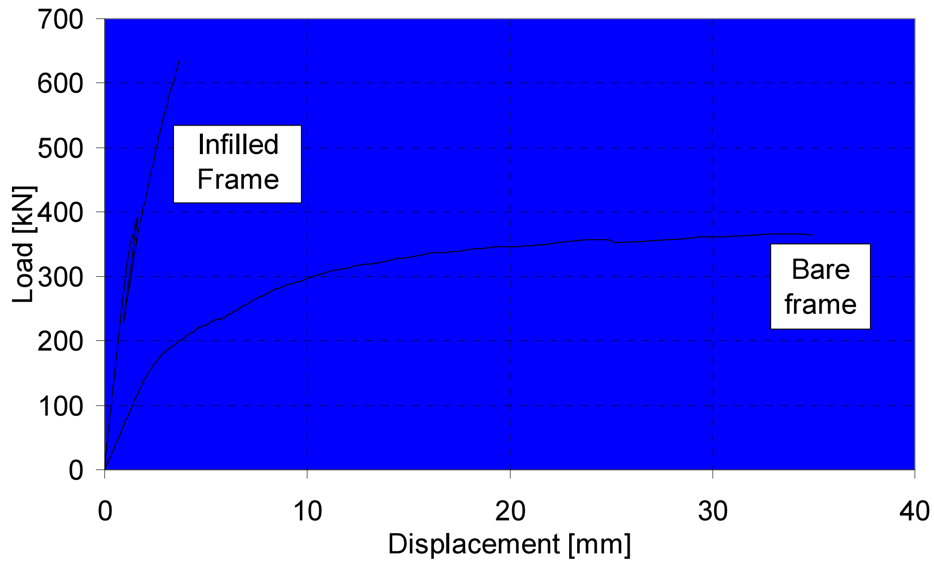
- Used to determine
  - deformation
  - development of damage
  - load displacement diagram

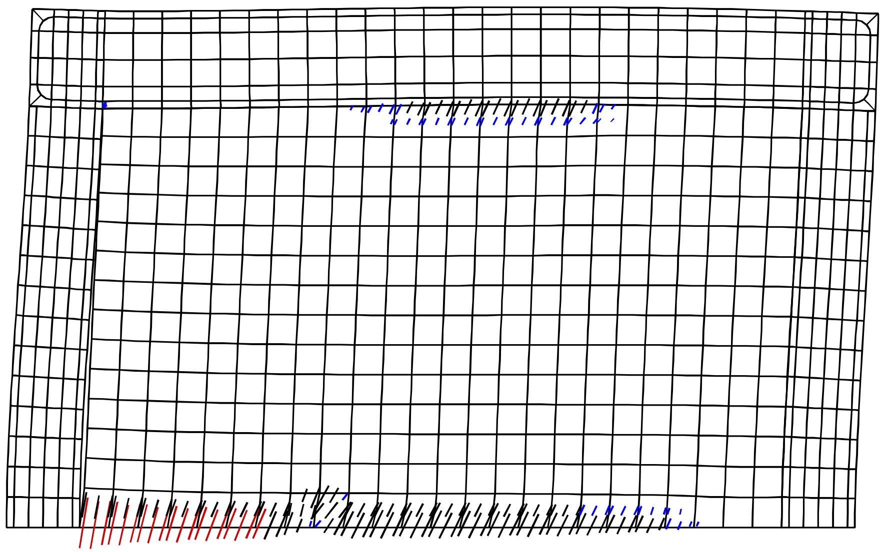


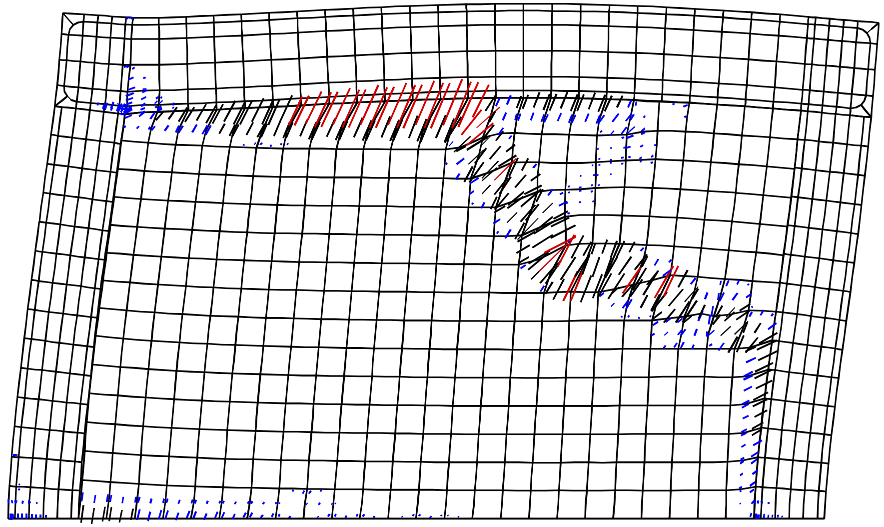
Difficulties related to F.E. analysis

- Correct prediction of structural response is difficult to obtain
  - numerical problems can be encountered especially beyond peak load
- Current non-linear material models are not complete particularly in the case of masonry
  - good agreement can be obtained only after experimental results are known

Analysis of an infilled r.c. frame

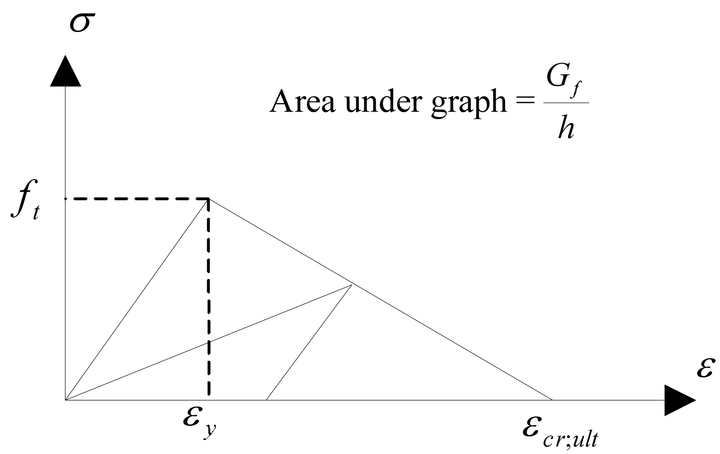






Material modelling issues

- brittle material response
- damage can occur in more than one location
- problem is path dependent and solutions are not necessarily unique



- limited insight into material behaviour and limitations in material model formulation can lead to mathematically ill-posed problems

Solution method

- Discretised problem leads to a set of non-linear equations which needs to be solved at each step of the analysis:

$$f : R^n \div R^n$$

find displacements such that $f = 0$

- System of equations is usually solved using the Newton method or variations thereof

$$J_k s_N = f_k$$

Objectives

- Formulation of material models
  - realistic response in tension, compression etc.
  - better numerical performance
- Formulation of solution methods which avoid the use of Newton iterations

Combined material model: cracking

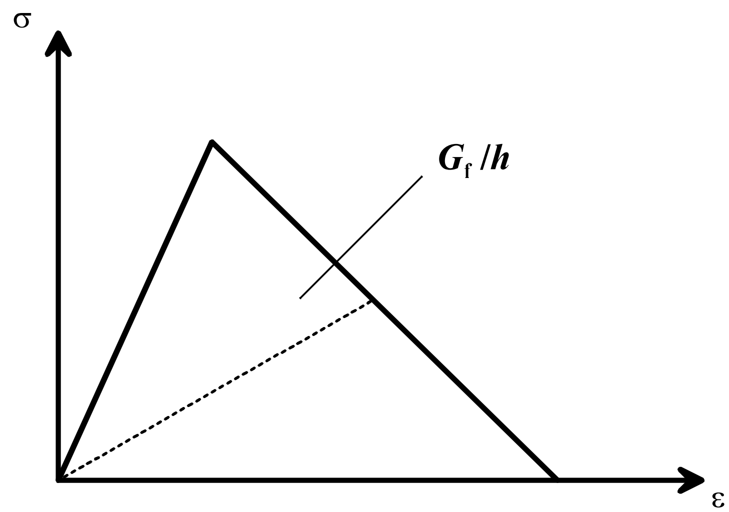
- Isotropic damage model:

$$\sigma = (1 - \omega)E\varepsilon$$

  – linear softening

  – ultimate strain

$$\varepsilon_{t,u} = \frac{2G_t}{f_t h}$$



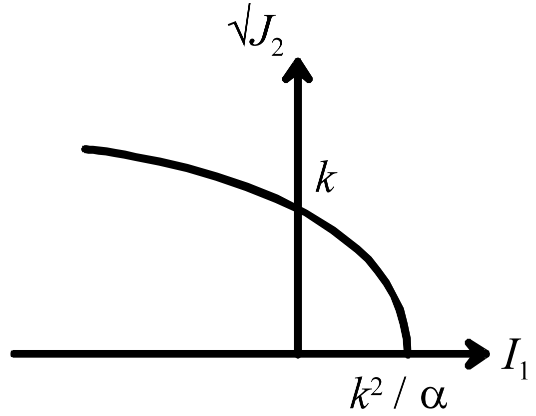
  – total stress

$$\sigma = E\left\{(1 - \omega)\begin{bmatrix} \max(\varepsilon_1, 0) \\ \max(\varepsilon_2, 0) \end{bmatrix} + \begin{bmatrix} \min(\varepsilon_1, 0) \\ \min(\varepsilon_2, 0) \end{bmatrix}\right\}$$

Crushing

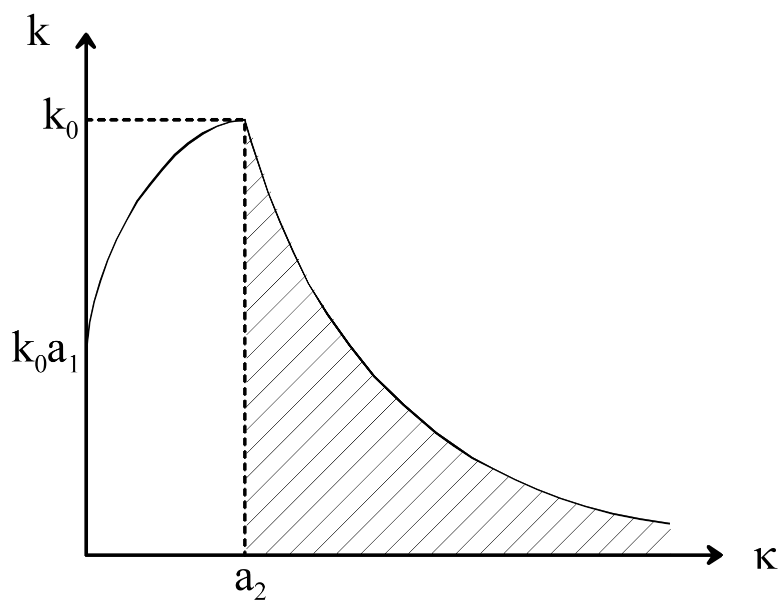
- Quadratic Drucker-Prager model

$$f \Uparrow \alpha \Rightarrow = J_2 + \vartheta I_1 \; ? \; k^2$$



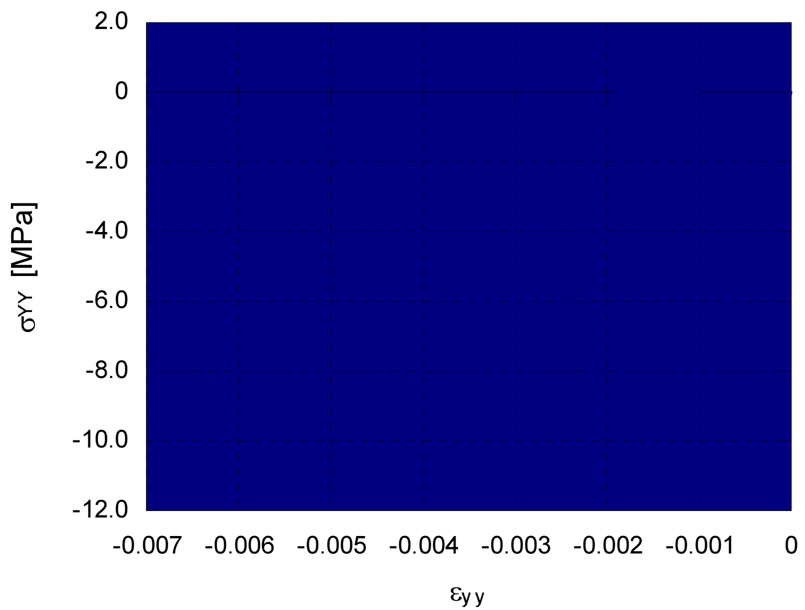
- damage

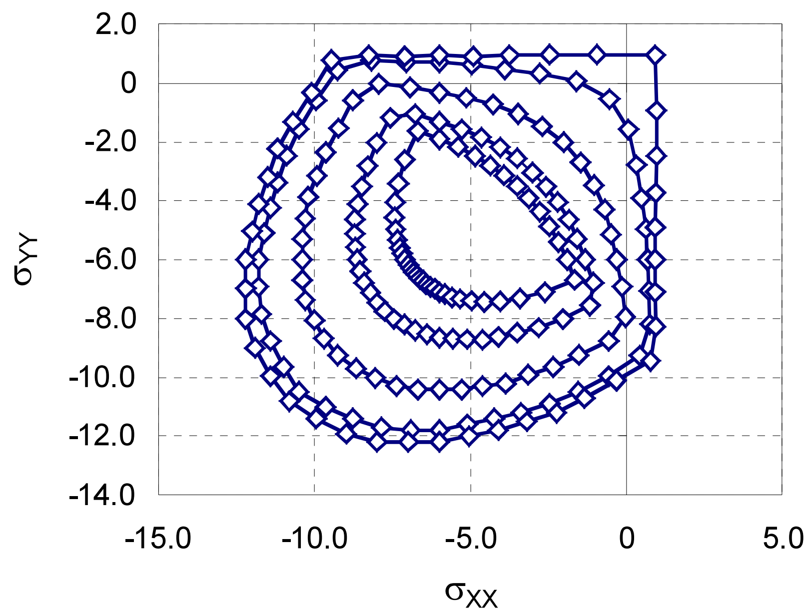
$$Y = Y_0 + \sqrt{\frac{2}{3} \mathrm{A\Pi} \, 6 \, \mathrm{A\Pi}}$$



Combined model response

- material with f_t=1 MPa; f_c=10 MPa





τ=0,1.5,3,4,5 MPa

Rate dependent damage model

- Based on the elastic damage model

$$\alpha = \Uparrow 1 \ ? \ N \Rightarrow E \Pi$$

  - damage rate law:

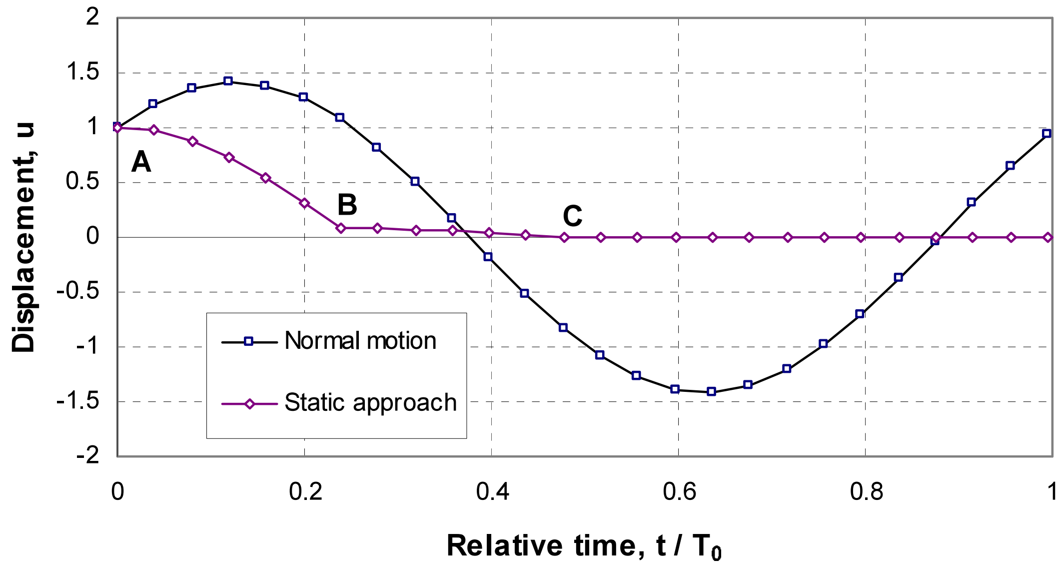
$$\frac{d\delta}{dt} = c \left\langle \frac{\varepsilon}{\varepsilon_t} - 1 \right\rangle^{a} \left(1 - \delta\right)^{b}$$

Solution procedures

- Method which avoid use of Newton iterations
- Dynamic problems
  - predictor-corrector methods for problems described by ordinary differential equations with given initial conditions, e.g. Newmark's method, central difference method (explicit)
- Static problems
  - dynamic relaxation method

Static solution

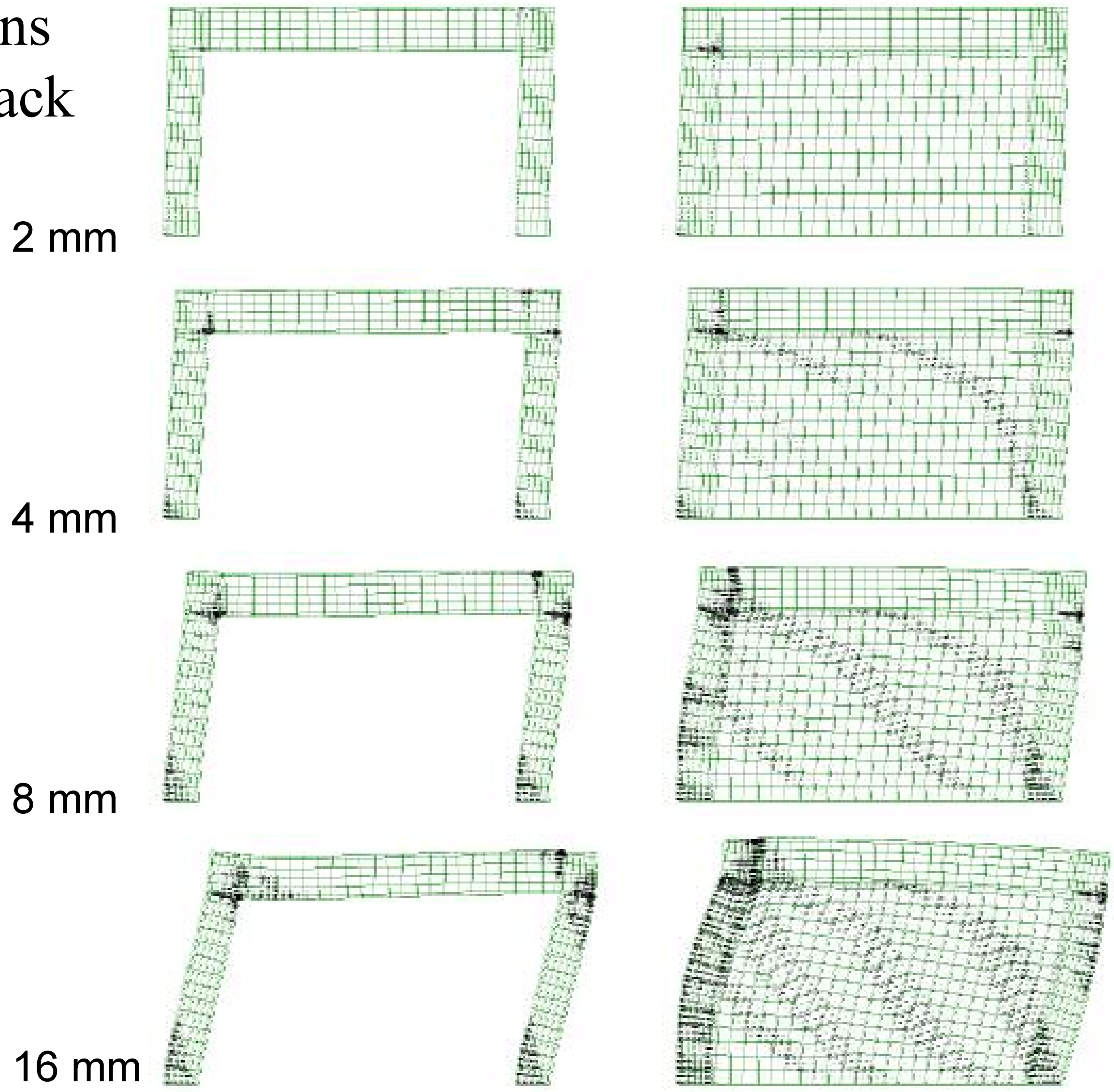
- Dynamic relaxation: introduces damping to make a transient explicit dynamic analysis converge to the static solution.
- Modified dynamic relaxation method for static analysis:
  - no damping required
  - velocity of system set to zero at moments when the absolute acceleration of the system has reached a minimum

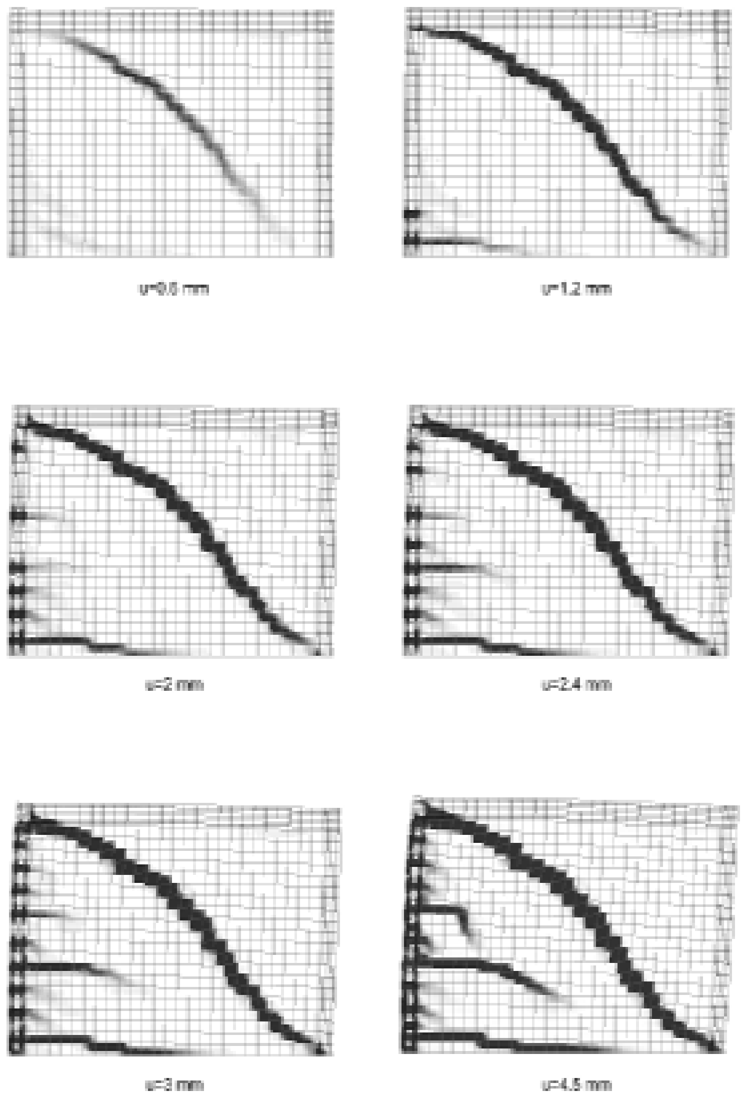


Problems considered

- Bar under tension to study problem of mesh independence
- Masonry panels under
  - tension
  - compression
  - horizontal loading
- Infilled r.c. frames

Deformations (x40) and crack patterns




u=0.6 mm
u=1.2 mm
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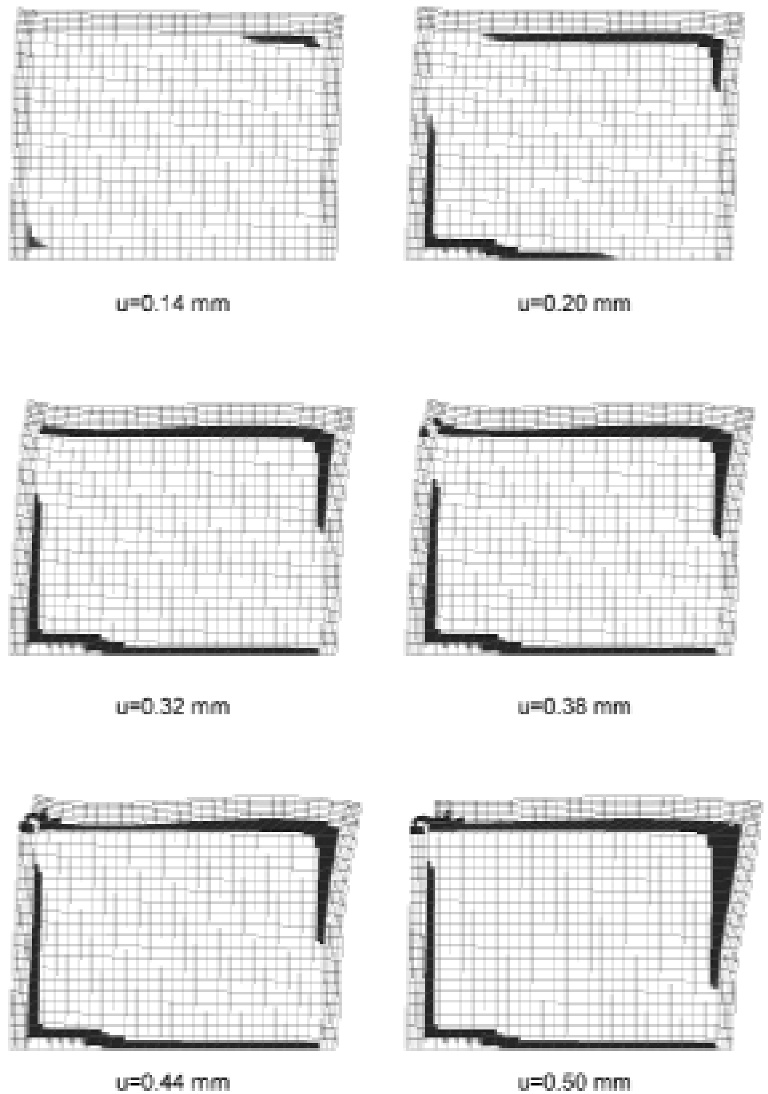
u=3 mm
u=4.5 mm


u=0.14 mm
u=0.20 mm
u=0.32 mm
u=0.38 mm
u=0.44 mm
u=0.50 mm

Conclusions

- Explicit dynamic analysis
  - requires no solution of non-linear equations
  - delivers results where other methods fail
  - allows a direct evaluation of material model performance
  - allows a simplification of the programming
- Additional DR type procedure can be used for a static analysis
  - automatic
  - delivers results similar to those obtainable with the Newton method with constant initial stiffness
  - some issues related to convergence speed and accuracy remain

Conclusions, continued

- Material models
  - marked interaction between cracking and crushing failure modes
  - shear behaviour has a great influence on the calculated global response
  - rate dependent model is superior to rate independent model in achieving mesh objective results

Possible future developments

- Solution methods
  - improvement of speed and accuracy
  - adaptive procedures
- Material models
  - comparison with experiments
  - calibration of model parameters
  - introduction of damping