Estimation of model parameters in nonlocal damage theories by inverse analysis techniques

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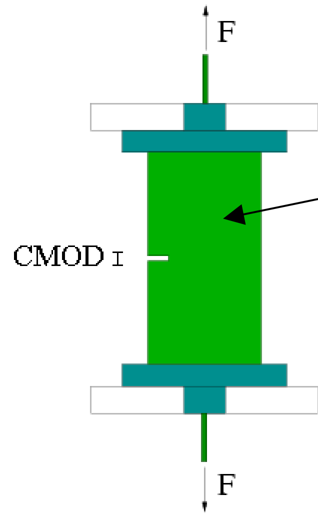
Outline

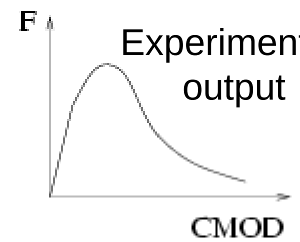
- ❑ Setting the scene
- ❑ Project objectives
- ❑ Forward problem
- ❑ Inverse problem
- ❑ The inverse techniques
- ❑ Experimental data
- ❑ Results
- ❑ Conclusions

Setting the scene

Experimental world



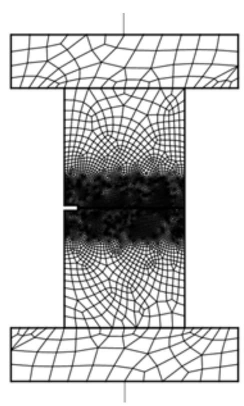
concrete specimen



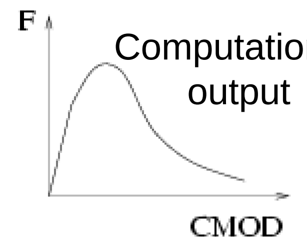
Experimental output

limited amount of work

Numerical world



mathematical equations containing parameters



Computational output

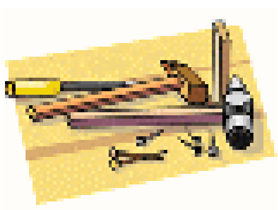
Setting the scene

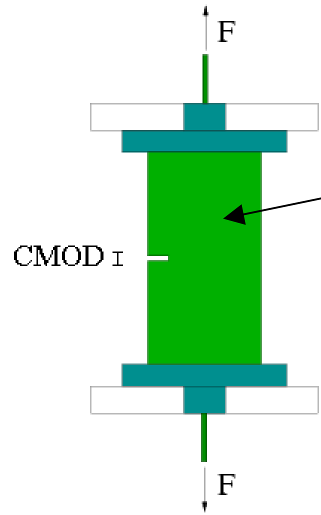
Result:

"Quit moving!"

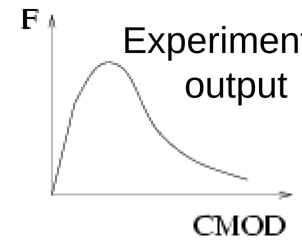
Setting the scene

Experimental world



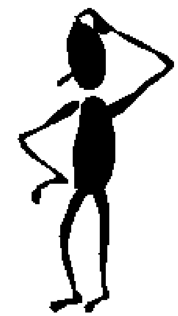


concrete specimen

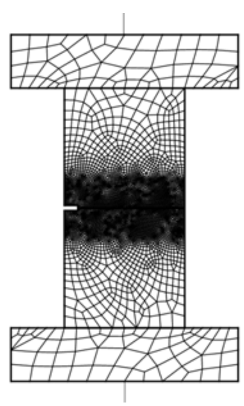


Experimental output

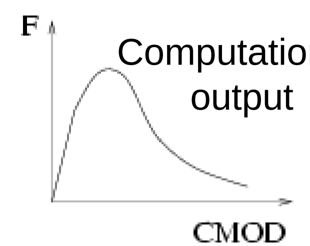
my PhD project



Numerical world



mathematical equations containing parameters



Computational output

Project objectives

❑ Parameters identification of continuum damage models:

- **Study of the aspects related to the solution of the inverse problem:**
  - ✓ Uniqueness and robustness of the solution (well-posedness of the inverse problem)
  - ✓ Factors of influence for the solution (e.g. experimental uncertainty, initial guess)
  - ✓ Qualitative and quantitative choice of the experimental data

- **Study of the aspects related to the choice of the inverse technique (best strategy)**
  - ✓ Effectiveness (how close to the solution)
  - ✓ Efficiency (time)
  - ✓ Reliability

Project objectives

- Insight in the calibrated numerical model:

  - solving the inverse problem needs insight in the forward problem, otherwise it reduces to mere data fitting
  - solving the inverse problem helps to have insight in the forward problem (e.g. length scale)
  - Investigation of the limitations of applicability, reliability and predictive capabilities of the calibrated numerical model (size effect and geometry effect)

Project objectives

- **Study of the problem of objectively extracting intrinsic material properties from structural experimental responses:**

  - **Numerical model is an approximation of the reality**
    - many external factors that play a role in the laboratory tests are difficult to be identified, quantified and included in the model
    - acting only on the model parameters may not be sufficient to cover the approximation
    - consequence: not constant material parameters.

  - **Possible dependency of the material parameters from**
    - structural factors: the boundary conditions, the load conditions, the specimen size and geometry
    - environmental and manufacturing factors
    - time and/or deformation state

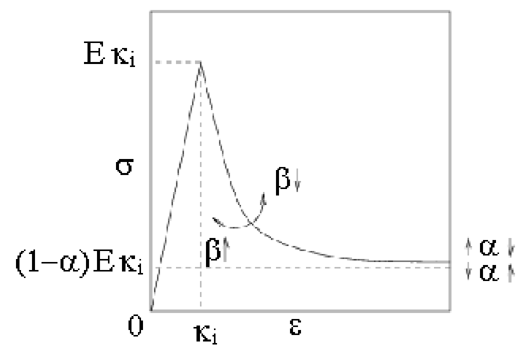
  - **Inverse problem only valid tool to link local law at the material point level with structural response**

The numerical model (forward problem)

Gradient-enhanced continuum damage model

$$\begin{cases} \boldsymbol{\sigma} = (1-\omega)\mathbf{D}^{el}\boldsymbol{\varepsilon} \\ \varepsilon_{eq} = \varepsilon_{eq}(\boldsymbol{\varepsilon}) \\ \omega = \omega(\kappa) \\ \kappa = max(\kappa_i, \bar{\varepsilon}_{eq}) \end{cases}$$

$$\left\{ \bar{\varepsilon}_{eq} - c\nabla^2 \bar{\varepsilon}_{eq} = \varepsilon_{eq} \right.$$



tensile-compressive strength ratio

gradient parameter (related to length scale)

$$\mathbf{x}^T = [\underbrace{E, \nu}, \kappa_i, \underbrace{\alpha, \beta}, \eta, c]$$

elastic parameters

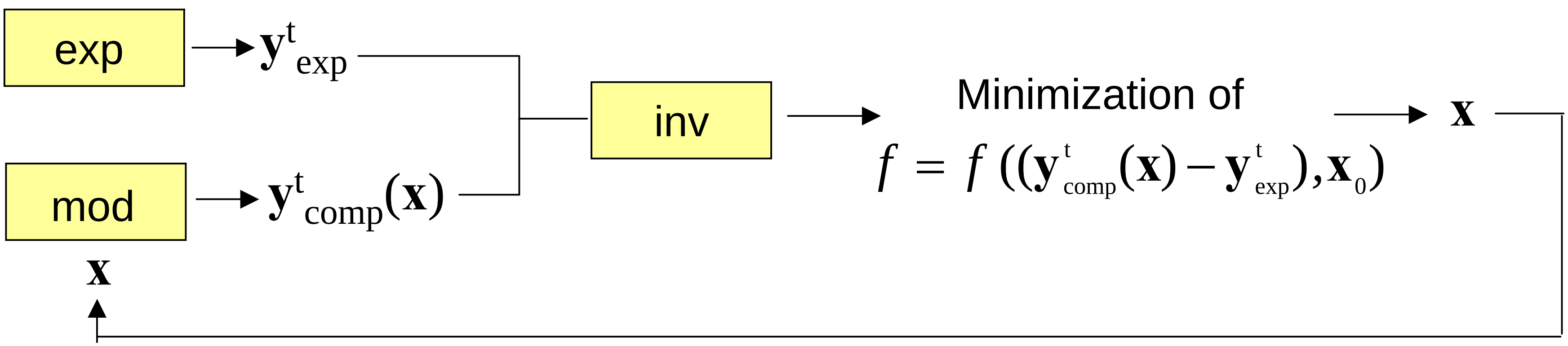
damage threshold

softening curve parameters

Simplification: $\mathbf{x}^T = [\alpha, \beta, c]$

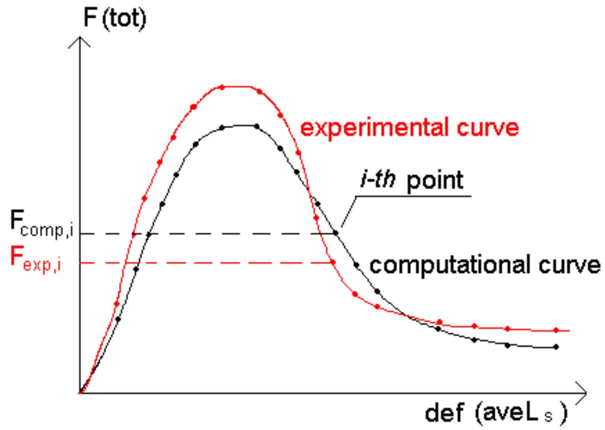
The Inverse Problem

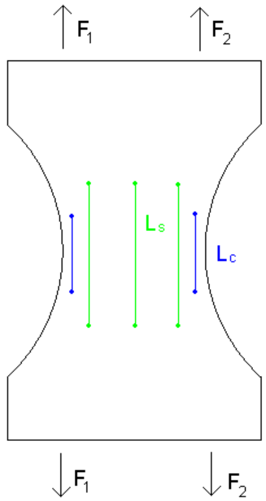
Minimization of an objective function



The Inverse Problem

Definition of the objective function





$$\mathbf{y}_{comp}(\mathbf{x}) = \left[F_{comp,1}(\mathbf{x}) \quad \dots \quad F_{comp,i}(\mathbf{x}) \quad \dots \quad F_{comp,N}(\mathbf{x})\right]^T$$

$$\mathbf{y}_{exp} = \left[F_{exp,1} \quad \dots \quad F_{exp,i} \quad \dots \quad F_{exp,N}\right]^T$$

mean

$$\mathbf{C}_{exp} = \begin{bmatrix} C^2_{exp,1} & 0 & 0 \\ 0 & C^2_{exp,i} & 0 \\ 0 & 0 & C^2_{exp,N} \end{bmatrix}$$

covariance

$$f(\mathbf{x}) = (\mathbf{y}_{comp}(\mathbf{x}) - \mathbf{y}_{exp})^T \cdot \mathbf{C}^{-1}_{exp} \cdot (\mathbf{y}_{comp}(\mathbf{x}) - \mathbf{y}_{exp}) = \sum_{i=1}^{N} \frac{1}{C^2_{exp,i}} (F_{comp,i}(\mathbf{x}) - F_{exp,i})^2$$

weighted squared distance between exp and comp vectors

The inverse techniques

- KNN method

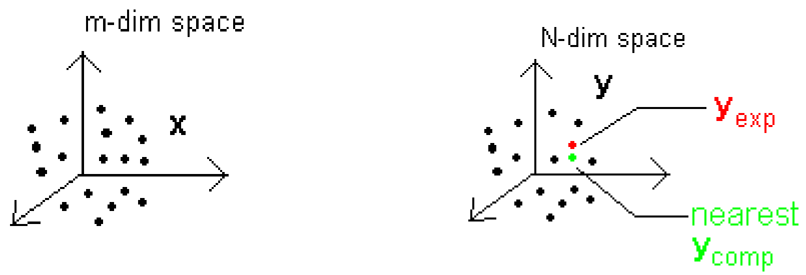
- Kalman filter method

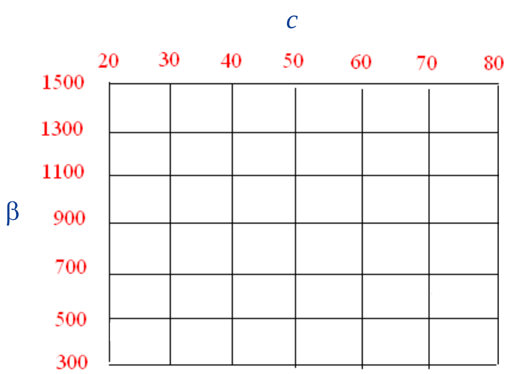
K-Nearest Neighbors method (KNN)

$$\hat{\mathbf{x}} = \min_{\mathbf{x}} f(\mathbf{x})$$

✶ choose a population of model parameters sets $\mathbf{x}_i$ (creation of a grid)

✷ compute (forward problem) $\mathbf{y}_{comp}(\mathbf{x}_i)$

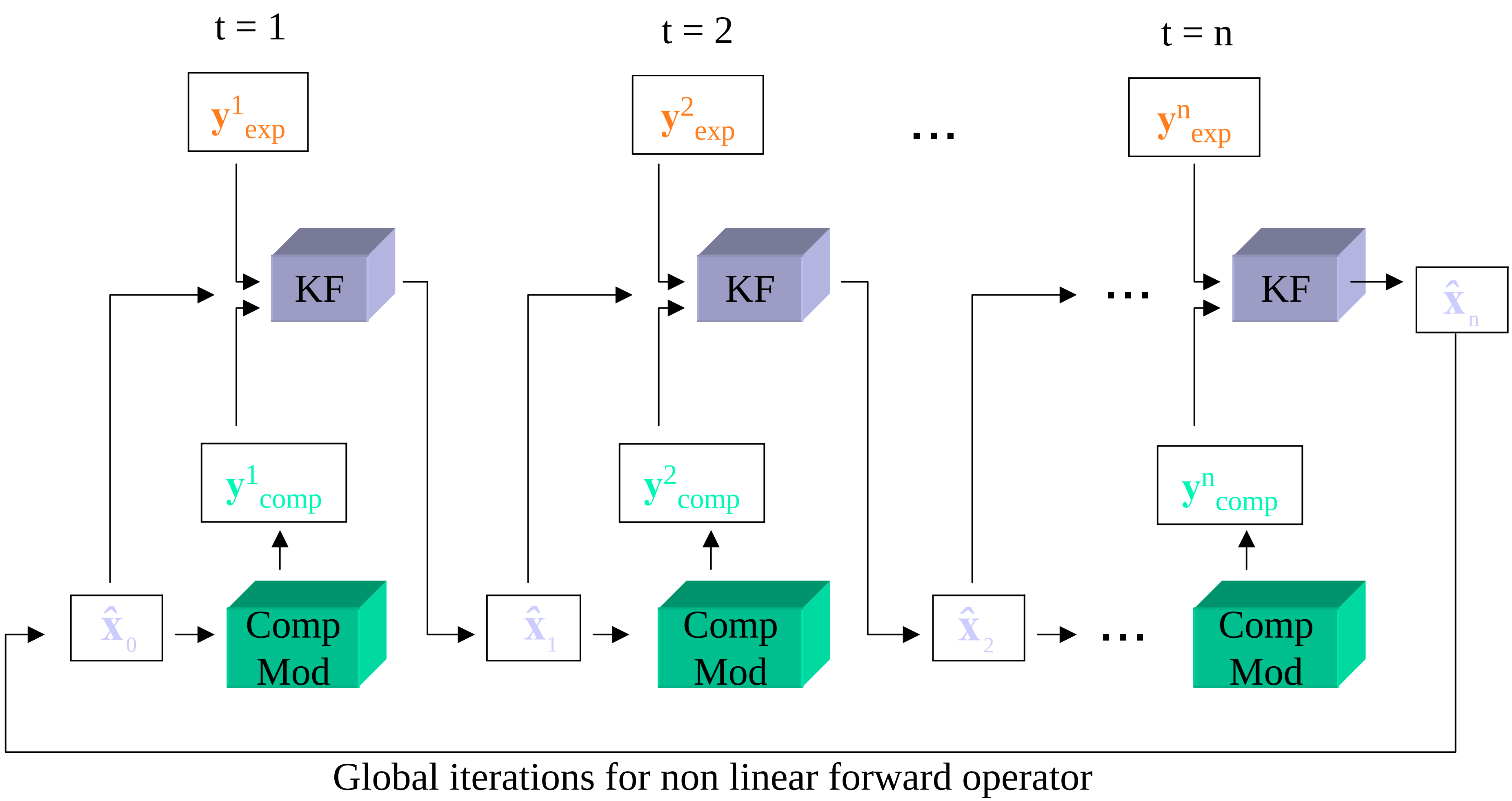




✸ evaluation of the weighted Euclidean distance $f(\mathbf{x}_i)$

❈ choose $\mathbf{x}$ that corresponds to the nearest neighbor of $\mathbf{y}_{exp}$ (K=1)

Kalman Filter method (KF)



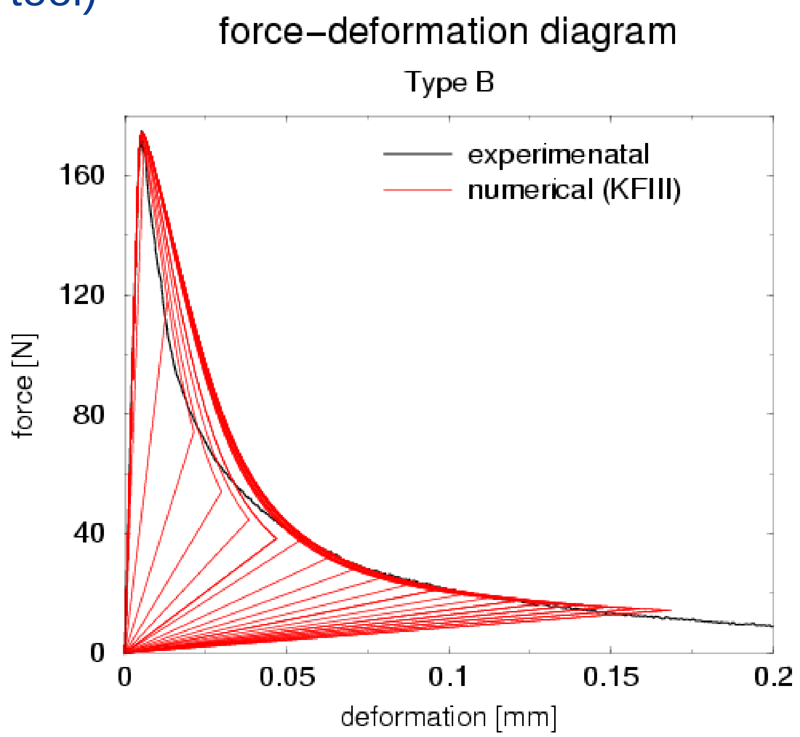
The inverse techniques

❑ KNN method

- ✓Derivative free method
- ✓General overview in the parameters space
- ✓Estimation of the initial guess
- ✓Parallel solutions of the forward problem
- ✓easily usable for any numerical model (external tool)

❑ Kalman filter method

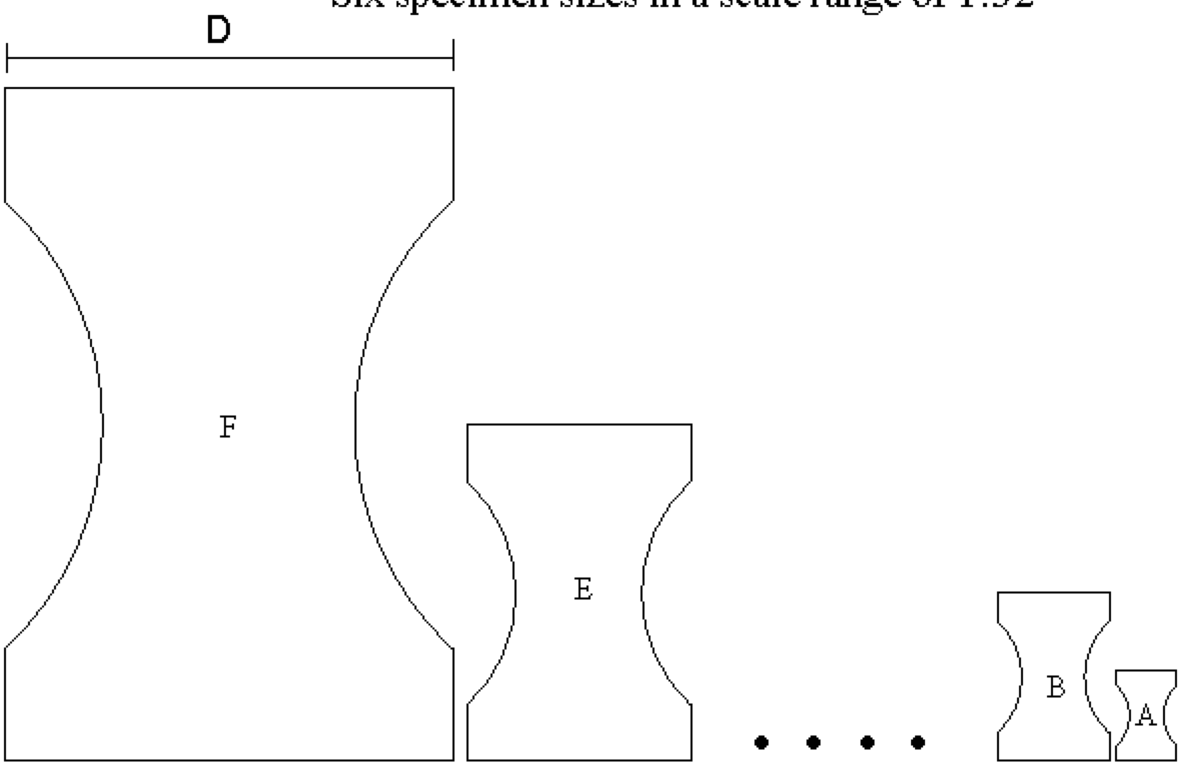
- ✓Refine the searching process
- ✓Parameters update during fracture process



Experimental data 1

Tensile size effect tests on dog-bone shaped specimens by van Vliet and van Mier (2000)

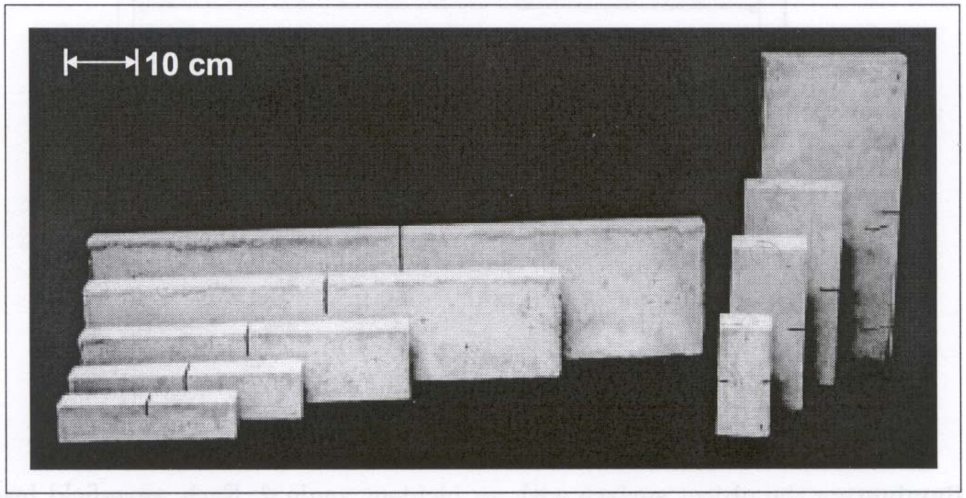
Six specimen sizes in a scale range of 1:32



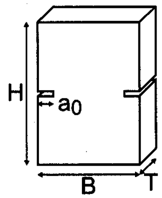
Experimental data 2

Tensile Size Effect Tests (different concrete mixes) by K. Hariri **(2000)**

- Three point bending tests (BG) on single-edge-notched concrete beams
- Uniaxial tensile tests (KG) on double-notched concrete prisms

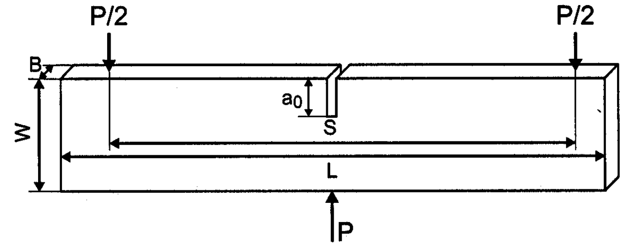


Double-edge notched tensile specimens



Size	Width B	Height H	Thickness T	Notch a_0
KG 1	80 mm	180 mm	80 mm	10 mm
KG 2	120 mm	270 mm	80 mm	15 mm
KG 3	160 mm	360 mm	80 mm	20 mm
KG 4	240 mm	540 mm	80 mm	30 mm

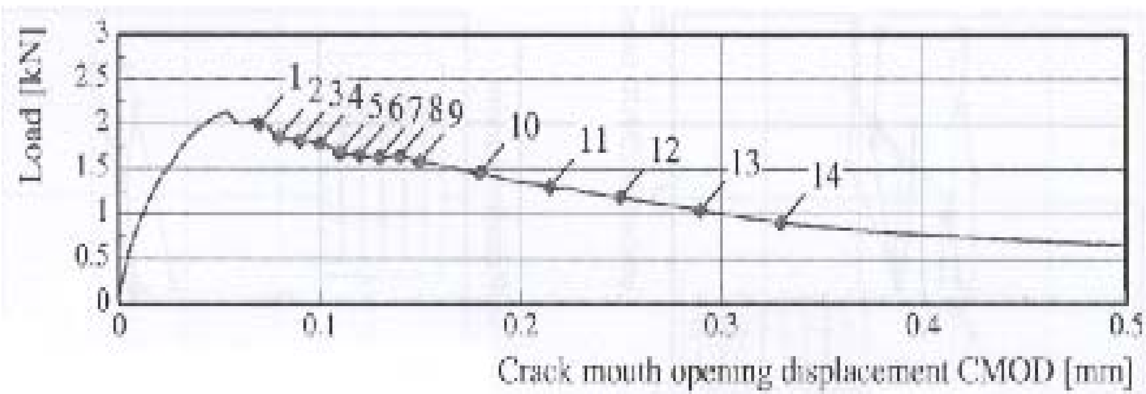
Single-edge notched bending specimens

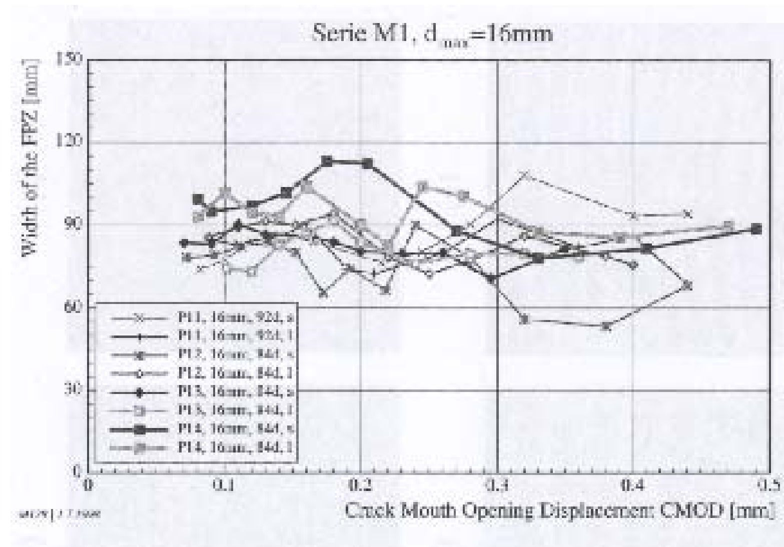


Size	Thickness B	Height W	Length L	Span S	Notch a_0
BG 1	60 mm	60 mm	260 mm	240 mm	20 mm
BG 2	60 mm	80 mm	380 mm	320 mm	30 mm
BG 3	60 mm	120 mm	560 mm	480 mm	40 mm
BG 4	60 mm	180 mm	840 mm	720 mm	60 mm
BG 5	60 mm	240 mm	1120 mm	960 mm	80 mm

Experimental data 2

- ❑ Experimental available results:





- ❑ Speckle Interferometry for the FPZ-size evaluation

Objective of the fitting

①Global curve one single size

②Global + local curve one single size

③Size effect curve (only peaks)

④Global curves different sizes

⑤Global + local curves different sizes

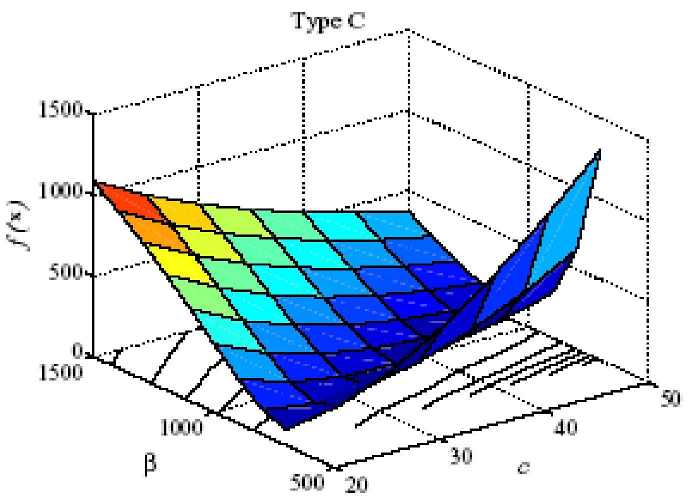
⑥Global + local curves different sizes and geometry

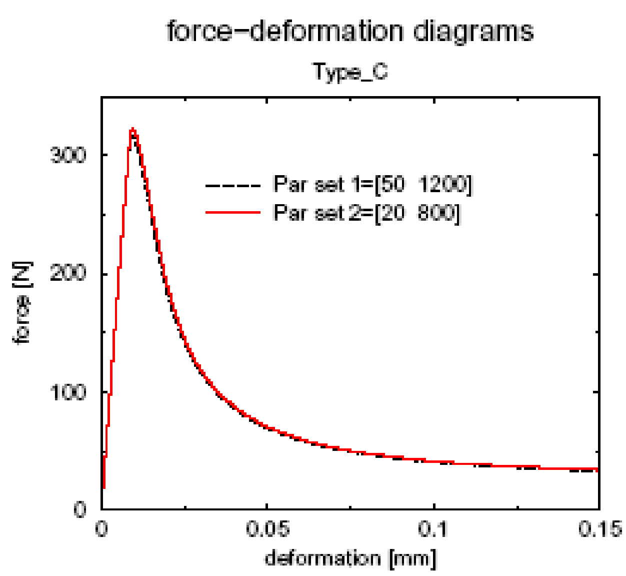
Results

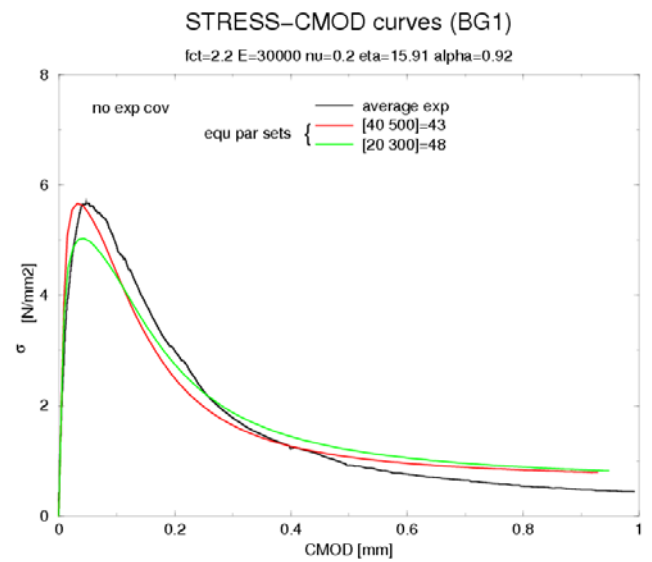
① Global curve one single size

✓Ill-posed inverse problem:

- not unique parameters set
- *c* and β correlated

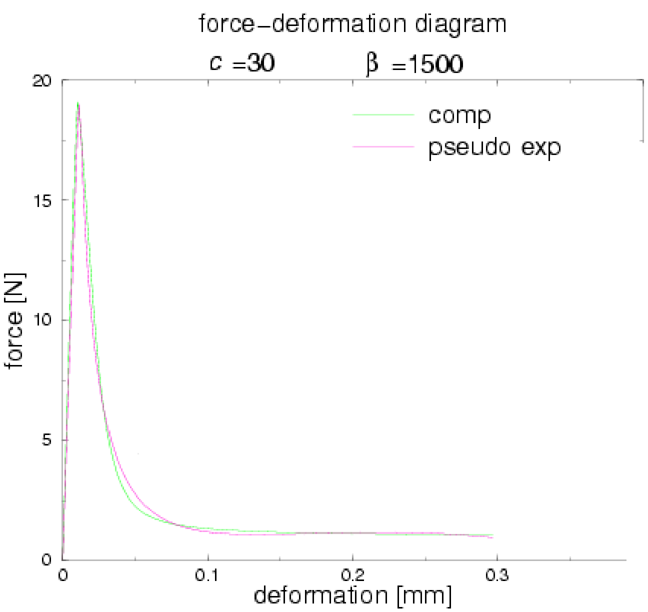


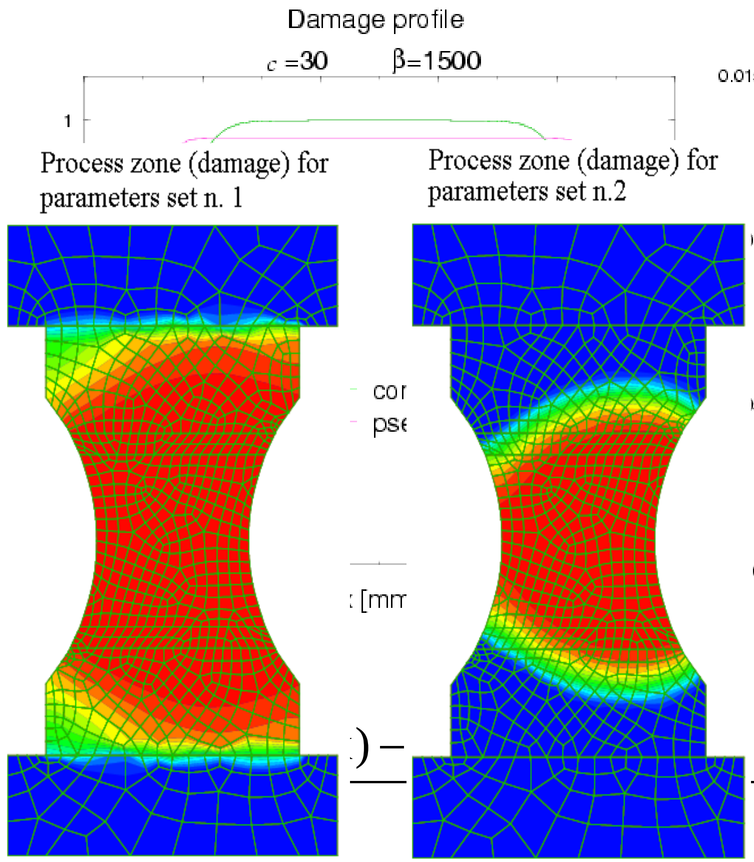


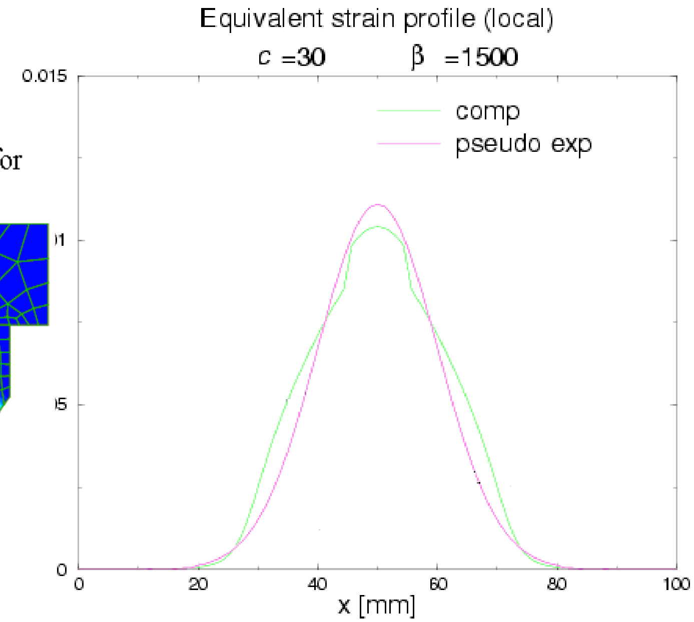


Results

② Global + local curve one single size





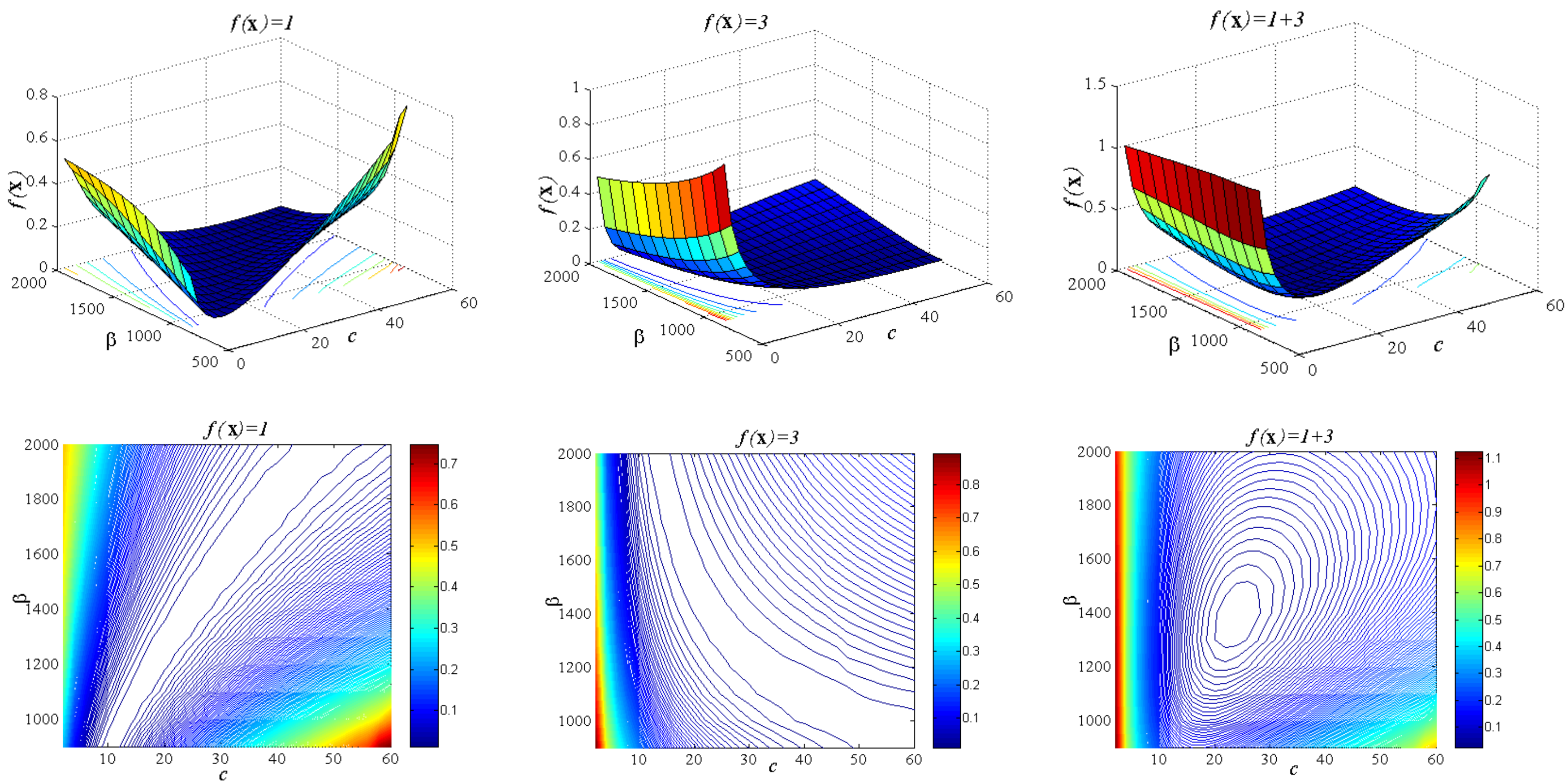


$$f(\mathbf{x}) = \overbrace{\frac{(\mathbf{F}_{comp}(\mathbf{x}) - \mathbf{F}_{exp})^T (\mathbf{F}_{comp}(\cdots}{\mathbf{F}_{exp}{}^T \mathbf{F}_{exp}}}^{1} \cdots \underbrace{\cdots) - \cdots \boldsymbol{\omega}_{exp})}_{2}$$

$$+ \underbrace{\frac{(\boldsymbol{\varepsilon}_{comp}(\mathbf{x}) - \boldsymbol{\varepsilon}_{exp})^T (\boldsymbol{\varepsilon}_{comp}(\mathbf{x}) - \boldsymbol{\varepsilon}_{exp})}{\boldsymbol{\varepsilon}_{exp}{}^T \boldsymbol{\varepsilon}_{exp}}}_{3}$$

Results

- ✓Single parameters set identified
- ✓Fitting of other sizes curves not guarantied



Results

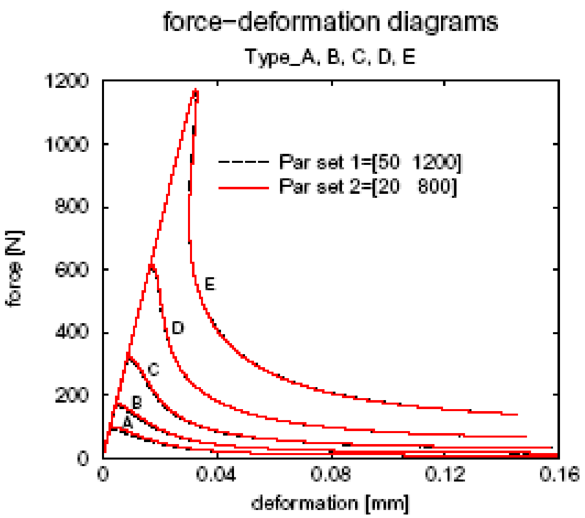
③ Size effect curve (only peaks)

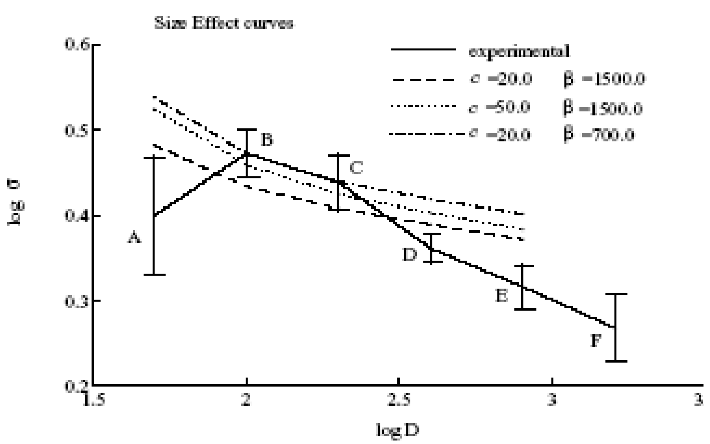
✓Different parameter sets could give “good” average fitting

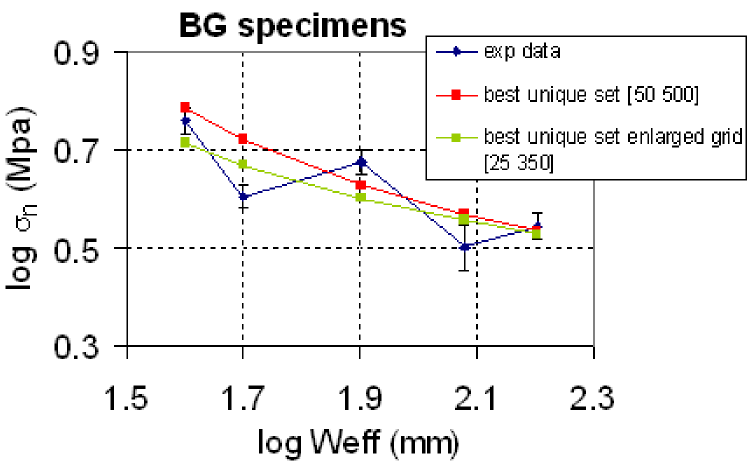
✓Fitting of the entire global curves not guarantied

✓No unique parameters set reproduces the real size effect curve (statistical effects not captured by the deterministic model)

✓The length scale may be used as tuner parameter.







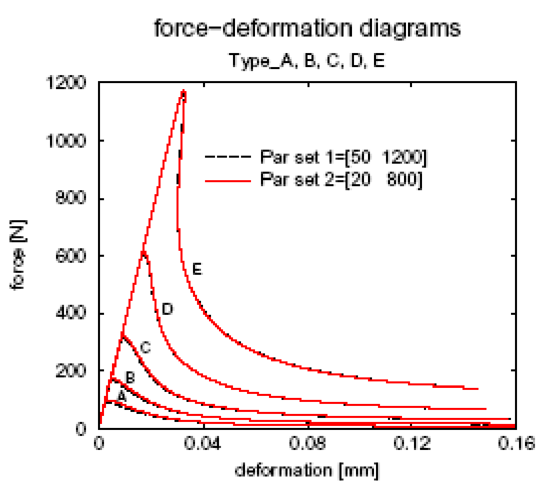
Results

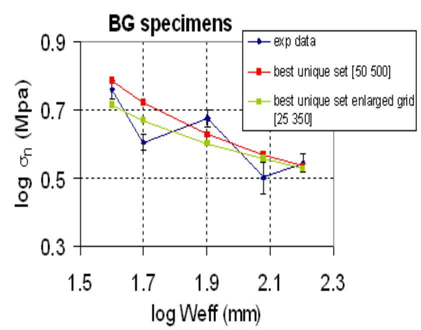
④ Global curves different sizes

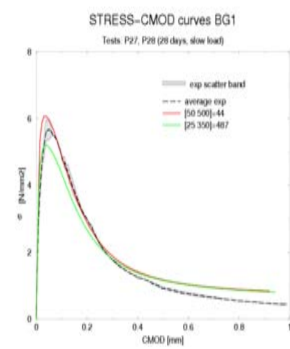
✓Different parameter sets could give “good” average fitting.

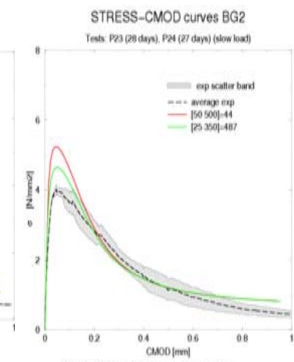
✓No unique parameters set reproduces the real size effect curve

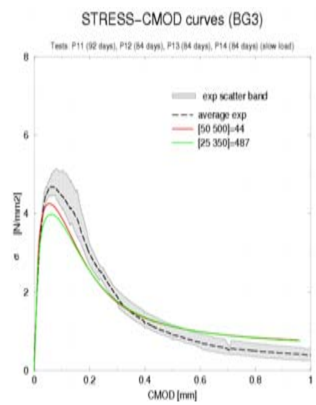
✓The length scale may be used as tuner parameter.

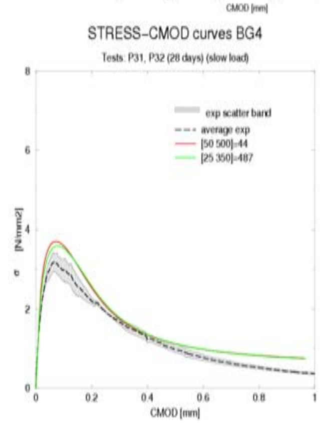


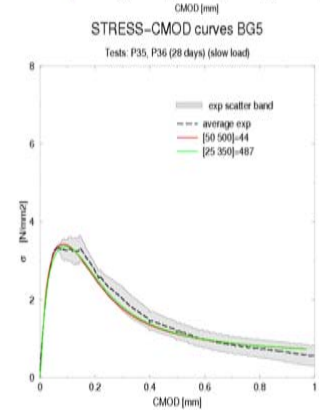












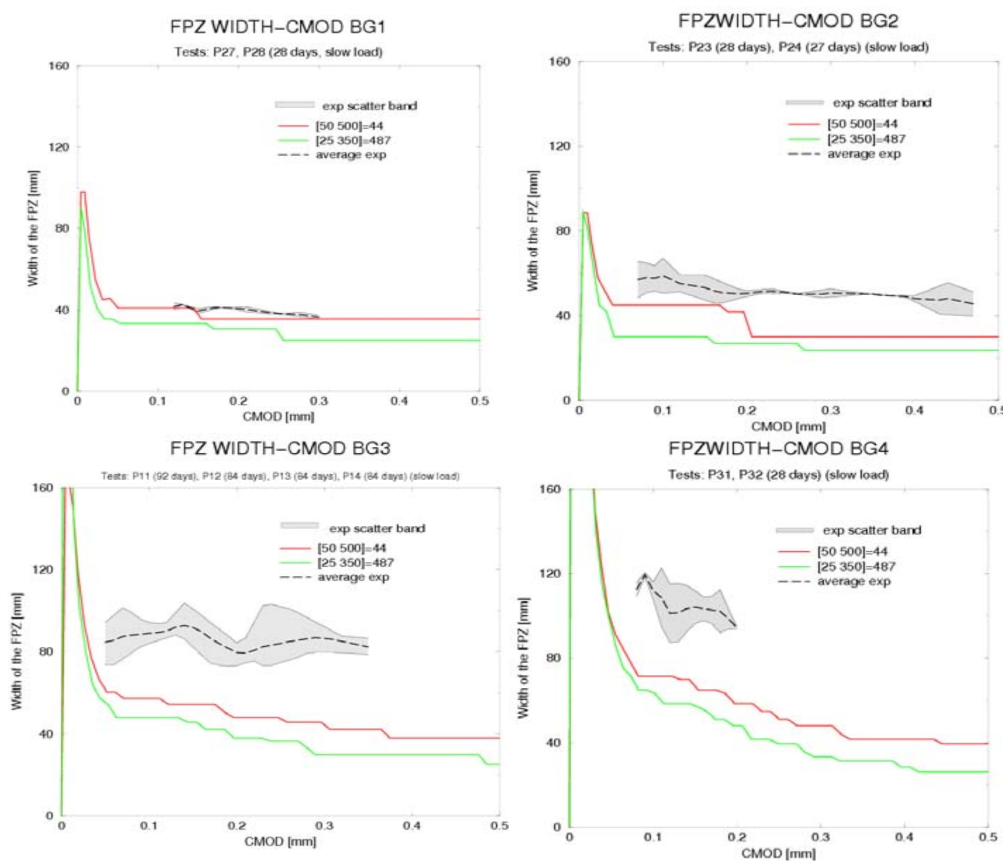
Results

Global + local curves different sizes

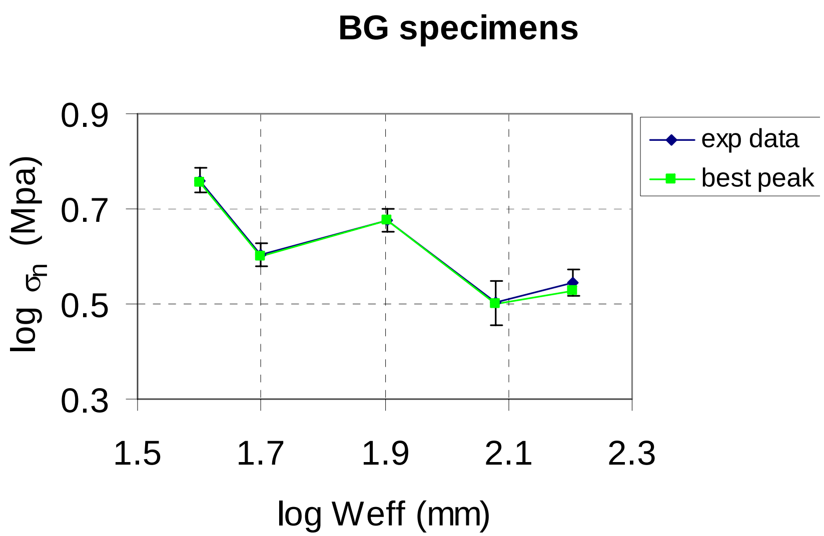
✓Single parameters set may be identified.

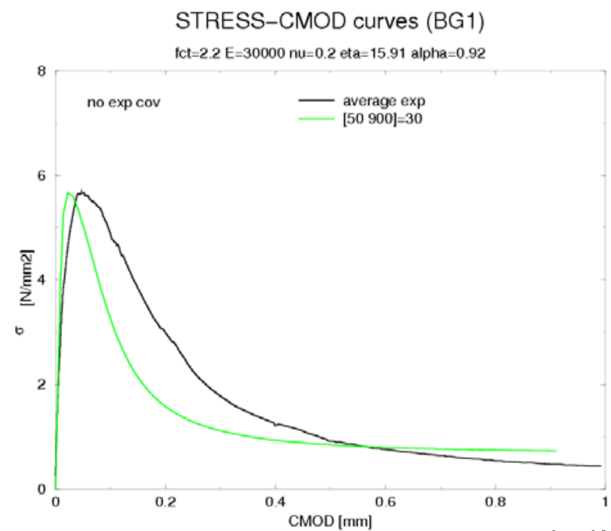
✓No unique parameters set reproduces the real size effect curve.

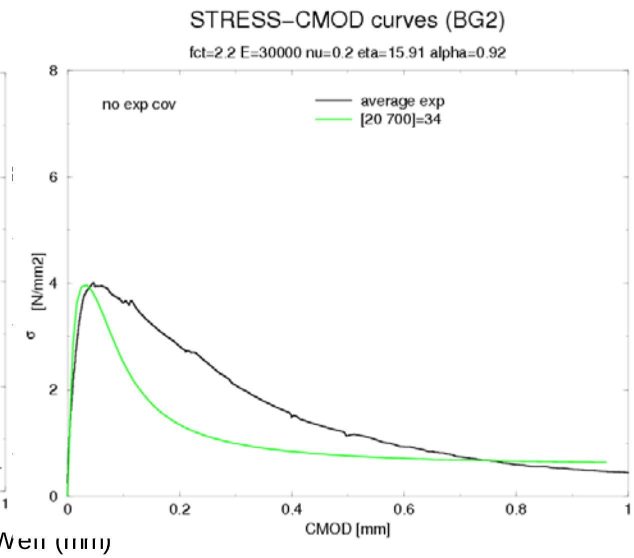
✓The length scale may be used as tuner parameter.

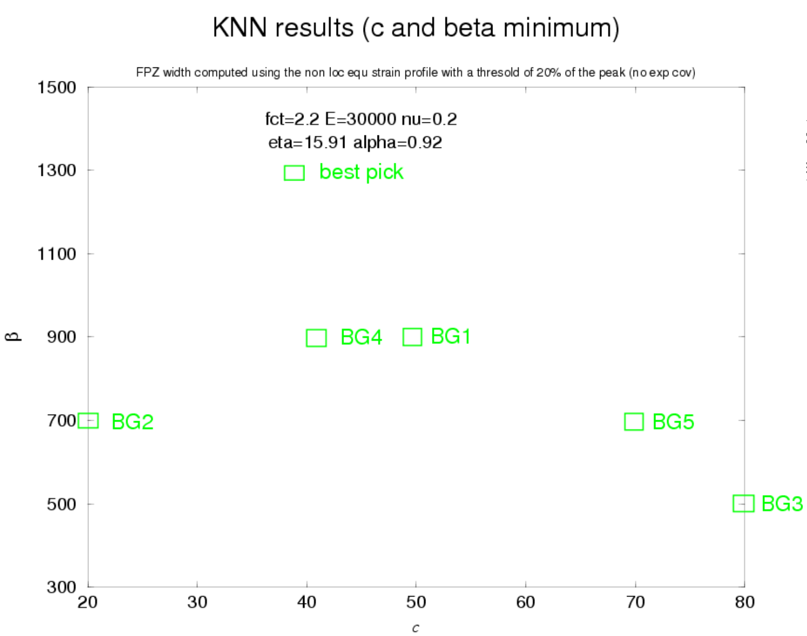


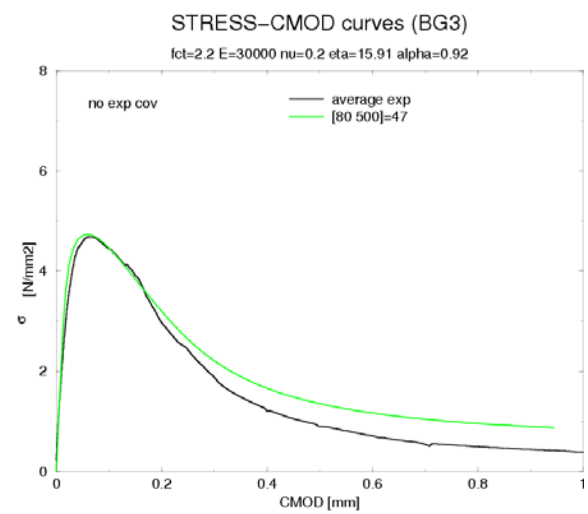
Results: no unique parameters set reproduces the real size effect curve.

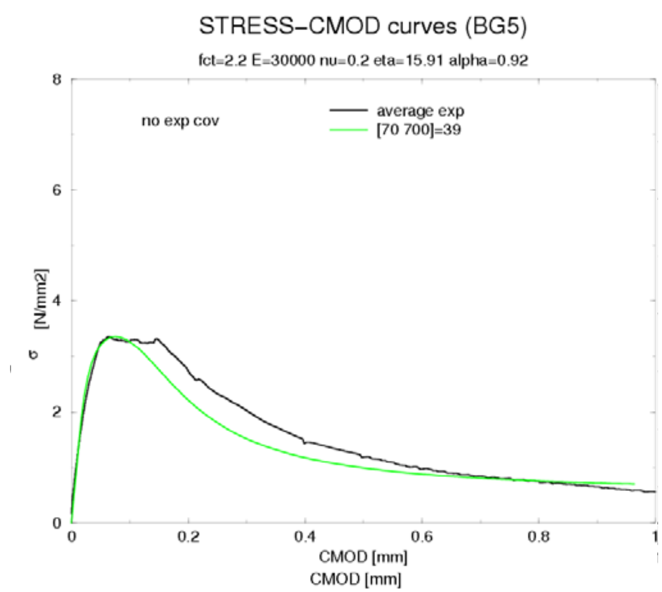






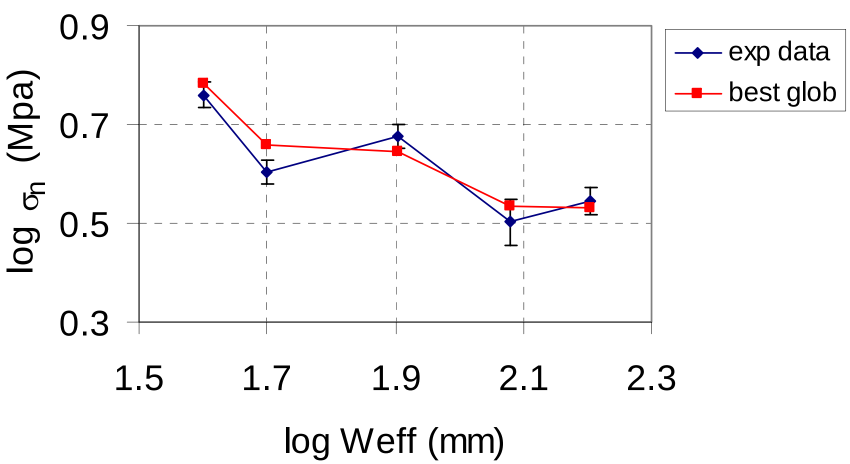


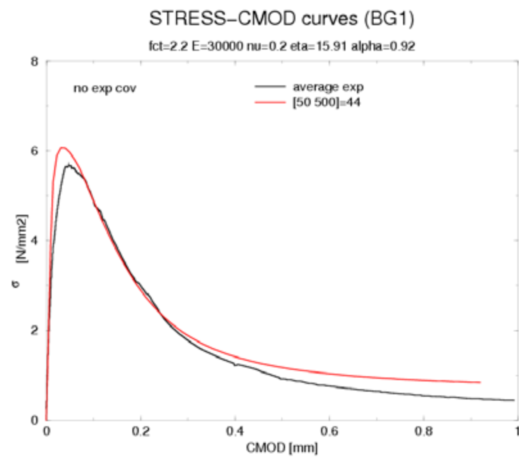


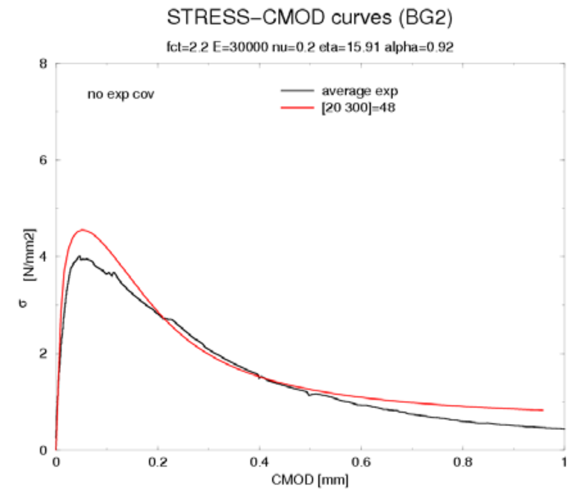


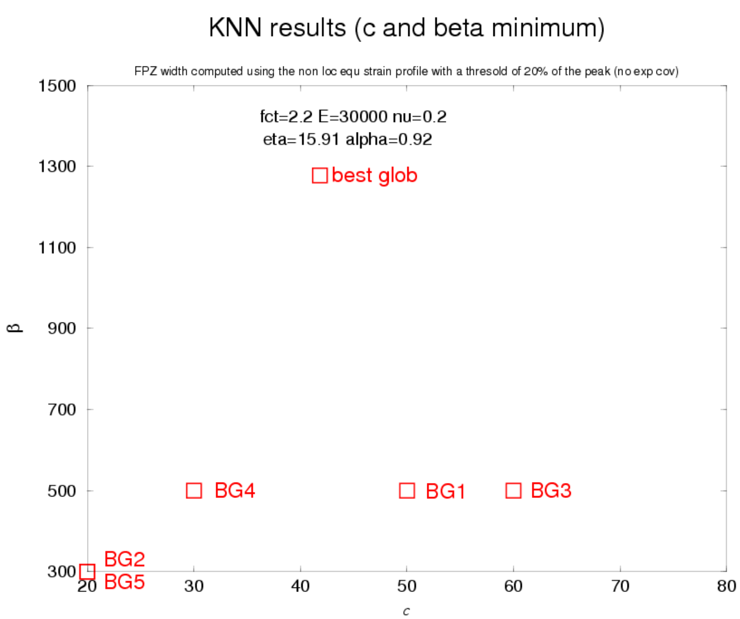
Results: fitting only the peaks ≠ fitting the entire global curves

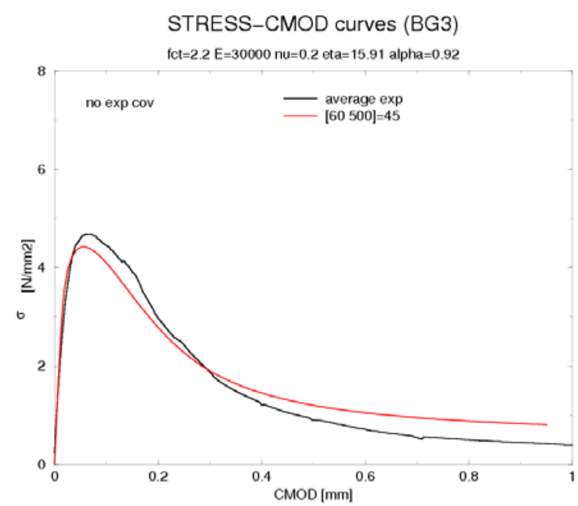
BG specimens

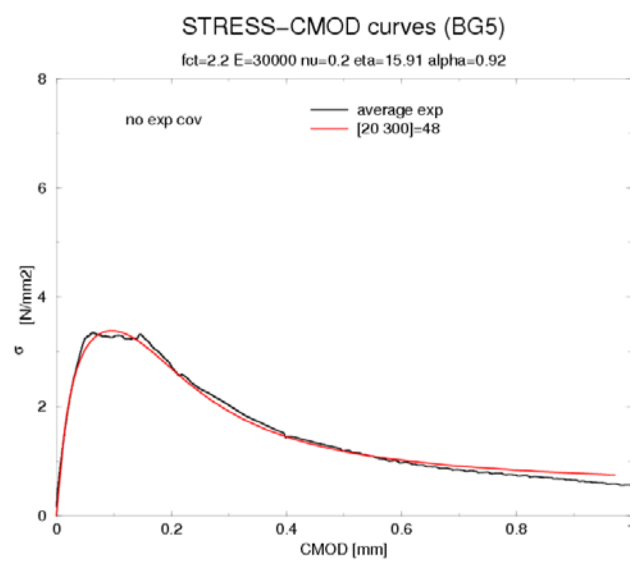




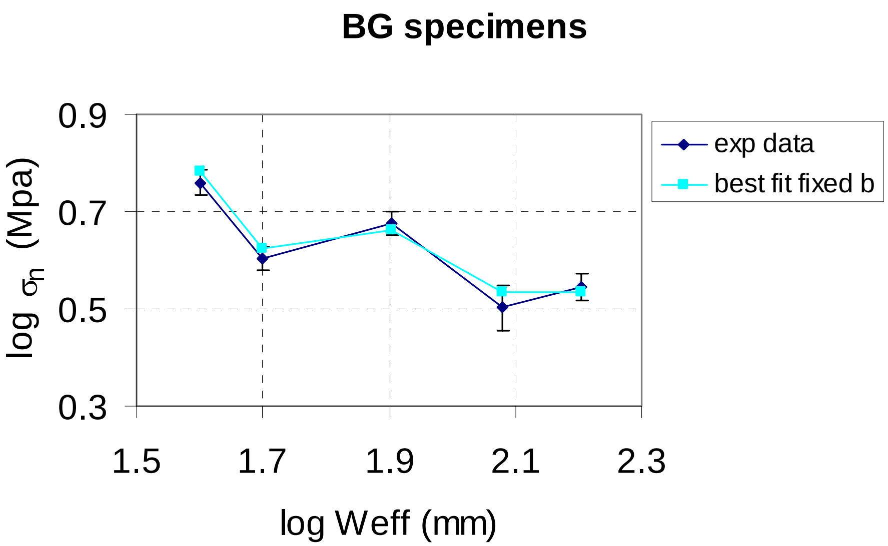


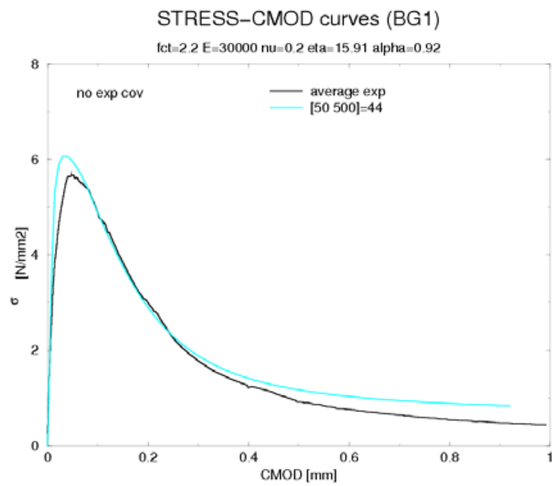


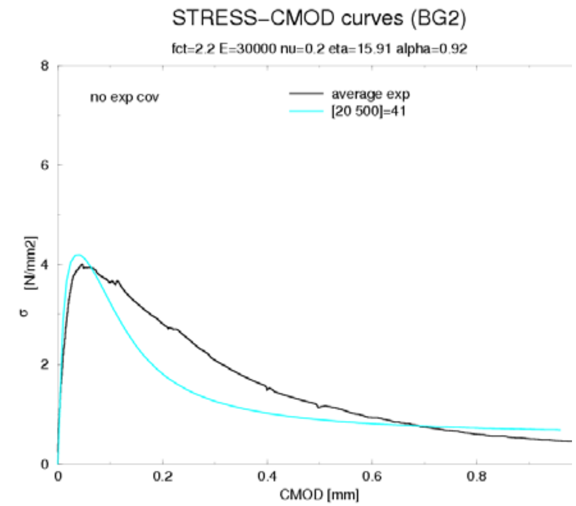


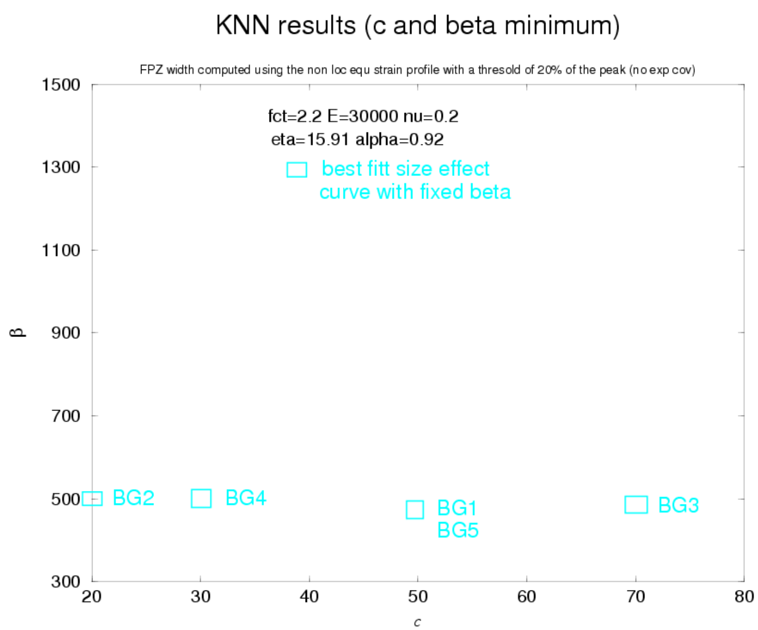


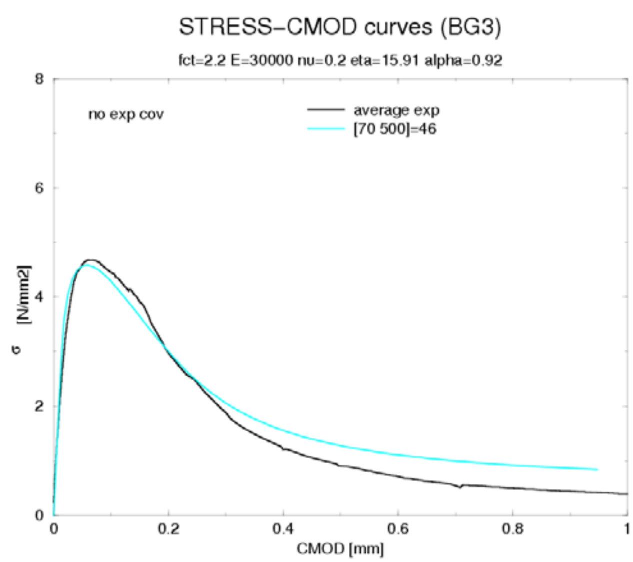
Results: the length scale may be used as tuner parameter

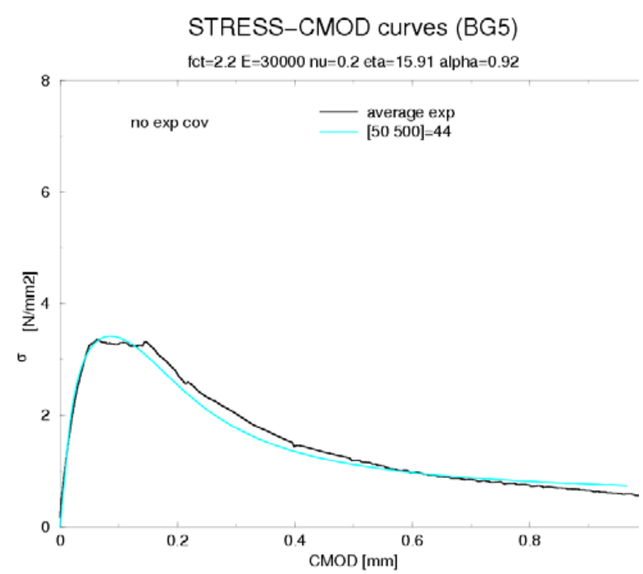








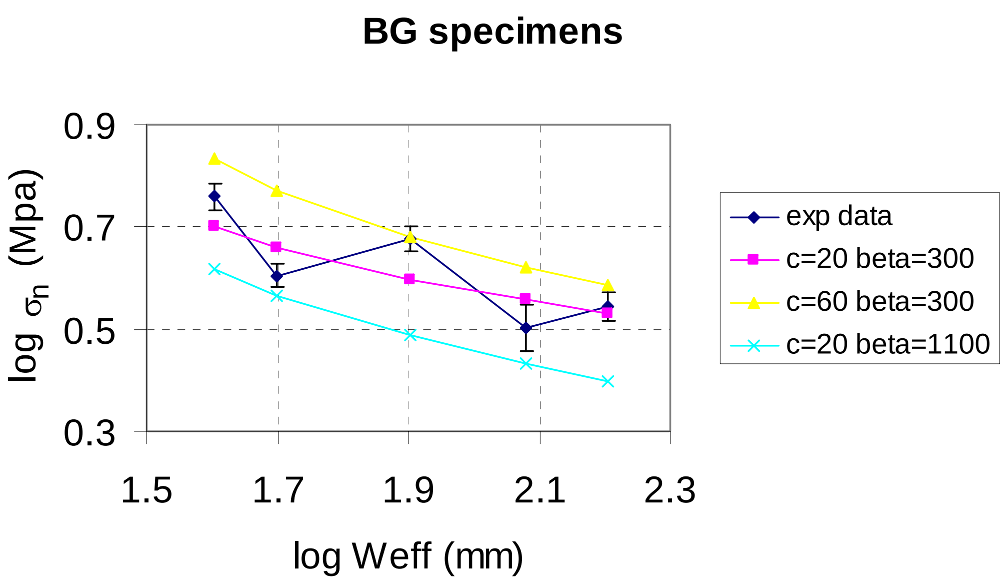


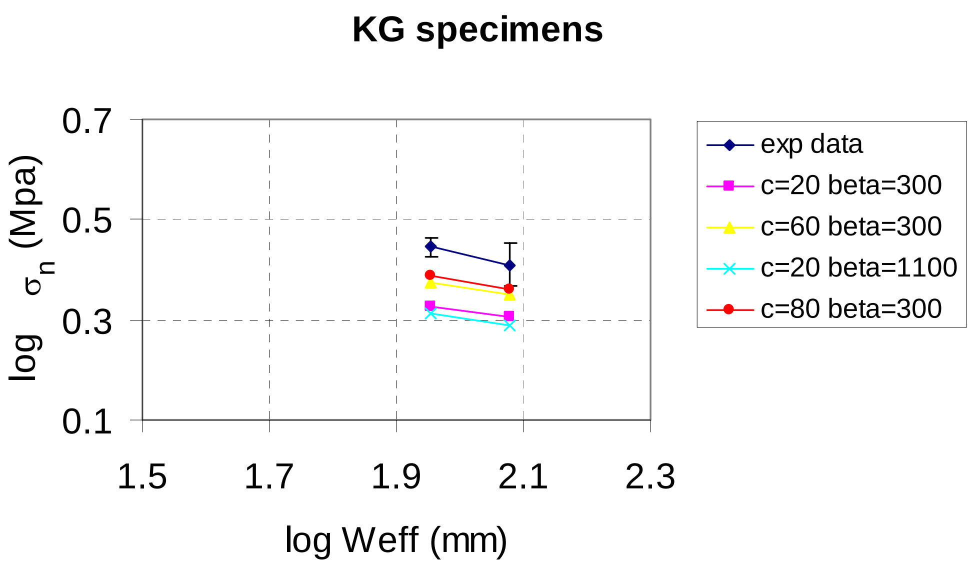


Results

Global + local curves different sizes and geometry

✓Structural effect





Conclusions (parameters identification strategy)

Parameters identification procedure (for β and *c*):

① KNN method to identify the best glob for each size.

② Find the better fixed βs (better representatives of the "best glob" population)

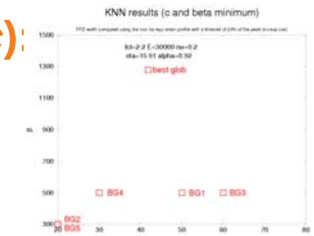
③ Find for each better β the best *c* for each size.

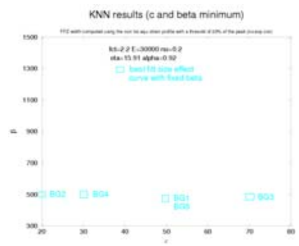
(peaks below the computational curve of a fixed [β *c*] couple have smaller *c* and vice versa)

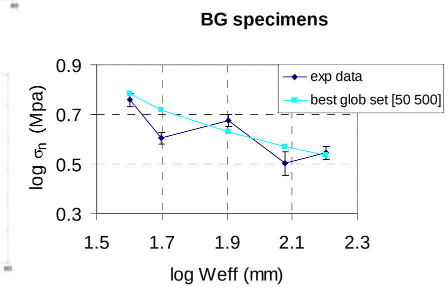
④ Find the best fixed *c* for each better β (the best representative of each "fixed β" population)

⑤ Find the best [*c*,β] set considering the local curves (priority of fitting to the glob curves)

Simply looking in the KNN matrix







⑥ "average" global fitting through all the sizes is satisfactory?

No → ⑦ Use the *c*s of step ③ for that fixed β (use *c* as tuner parameter)

Penalty of not a unique *c*

Yes → ⑦ Stop

⑧ KF on one par set for all the sizes

How the objective has to be fitted?

"average" global fitting through all the sizes is satisfactory?

No → ⑧ Go back to ⑥ for another [c,β] couple

Penalty of not fitting local data

No → ⑧ Use the best glob of step ①

⑨ KF on *c* and β for each size

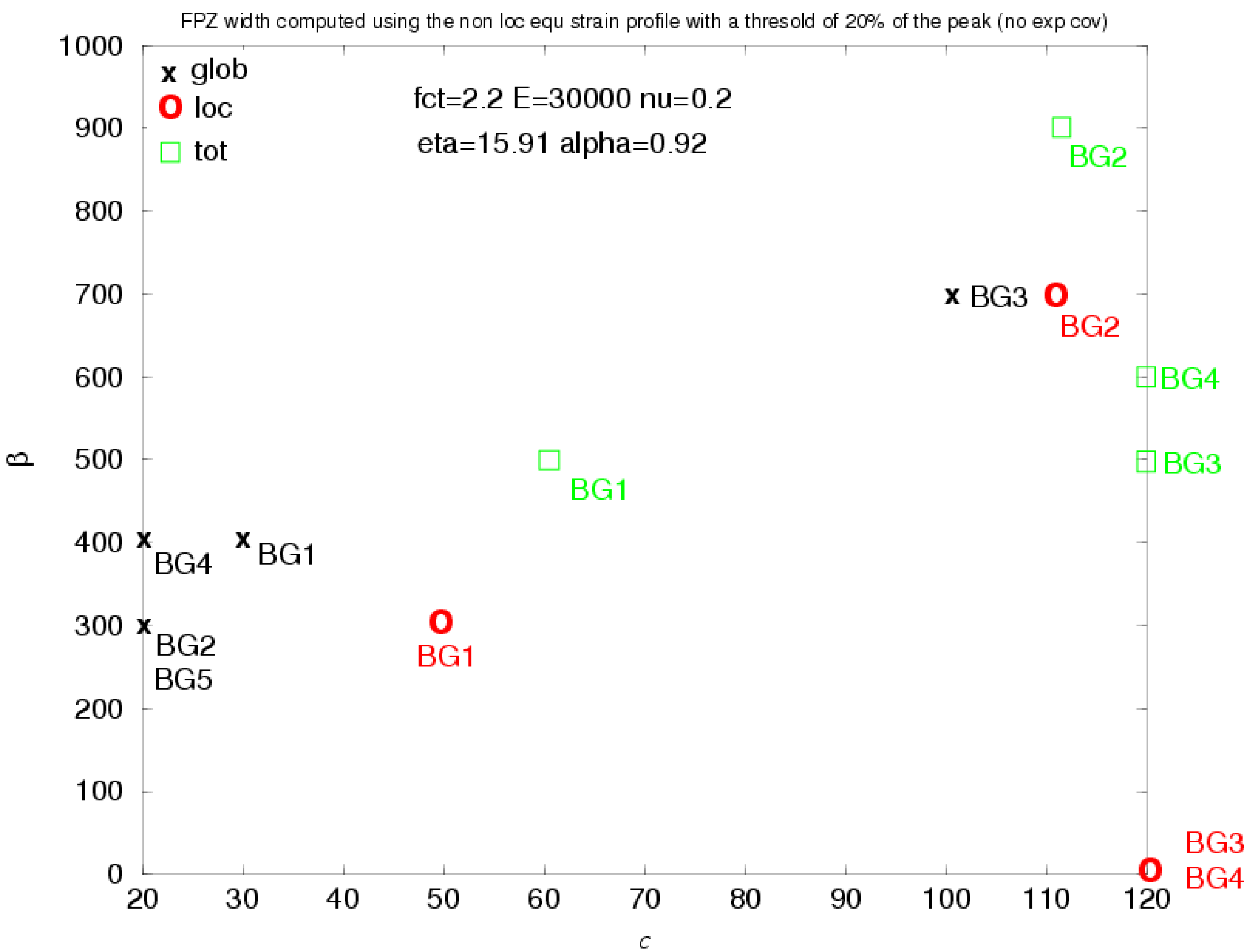
Penalty of not unique β

Yes → ⑧ Stop

⑨ KF on *c* for each size

Conclusions (Hariri tests: global overview)

KNN results (c and beta minimum)



- Average fitting of the global size effect obtained by one single set with *c* toward the smallest value.
- Detailed fitting of the global size effect varying *c*
- Spread of the parameters sets to obtain the best fitting of the local size effect.
- Best individuals at borders!!!
- Structural effect on the model parameters.
- May parameters identification, solved as inverse problem, completely substitute investigation at micro or meso-scale?

Additional slide 1