RESEARCH SCHOOL
INTEGRAL DESIGN OF STRUCTURES

Capturing complex non-linear structural response through a series of linear analyses

Supervisors: prof.dr.ir. Jan G. Rots | dr.ir. Max A. N. Hendriks

BACKGROUND

Reasons of research:

- Underground construction: existing buildings face the risk of being damaged due to imposed settlements
- Predicting crack patterns and crack widths is a difficult task because:
  - » making a representative model is complex
  - » obtaining a properly converged solution is difficult

BACKGROUND

Object of my research:

To develop and implement an accurate and robust analysis scheme – based on Sequentially Linear Analysis (SLA) – that can predict crack patterns and crack widths at building level.

OVERVIEW

- Introduction to Sequentially Linear Analysis (SLA)
- Adapted algorithm for non-proportional loading
- Example: analysis of an un-reinforced masonry façade subjected to tunnelling-induced settlements
- Current research: implementation of interface elements
- Conclusions and future work

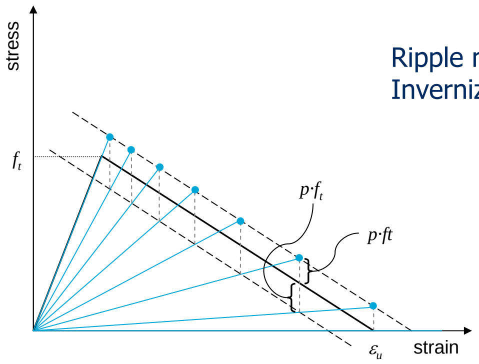
INTRODUCTION TO SEQUENTIALLY LINEAR ANALYSIS (SLA)

Event-by-event strategy by Rots & Invernizzi (2004).

Assumptions:

1. Material behaviour may be discretized by means of a "saw-tooth" model.

INTRODUCTION TO SEQUENTIALLY LINEAR ANALYSIS (SLA)



Ripple model by Rots, Belletti & Invernizzi (2006)

Note: contrary to regular damage models a finite number of damage states is being defined

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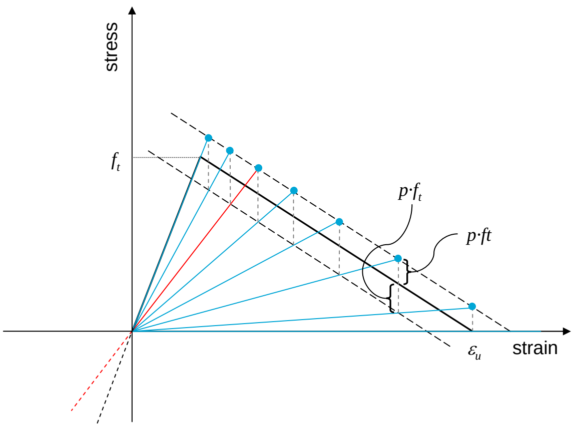

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Event-by-event strategy by Rots & Invernizzi (2004).

Assumptions:

1. Material behaviour may be discretized by means of a "saw-tooth" model.
2. The same secant stiffness is used for the tensile and compressive regime.
3. Per event a damage increment (i.e. an instantaneous change in stiffness) is applied to just *one* integration point.

INTRODUCTION TO SEQUENTIALLY LINEAR ANALYSIS (SLA)

Algorithm of a Sequentially Linear Analysis scheme:

1. Perform linear-elastic analysis with unit load.
2. Identify critical integration point.
3. Multiply unit load with critical load multiplier λ.
4. Apply instantaneous change in stiffness to critical integration point.
5. Return to step 1.

ADAPTED ALGORITHM FOR NON-PROPORTIONAL LOADING

Starting points/ assumptions:

- Any load can be attributed to either load set A (non-proportional loads) or load set B (proportional loads).
- Plane stress conditions, i.e. stress components:
  - » $\sigma_{xx;i}(\lambda) = \sigma_{xx;i;A} + \lambda\sigma_{xx;i;B}$
  - » $\sigma_{yy;i}(\lambda) = \sigma_{yy;i;A} + \lambda\sigma_{yy;i;B}$
  - » $\sigma_{xy;i}(\lambda) = \sigma_{xy;i;A} + \lambda\sigma_{xy;i;B}$

ADAPTED ALGORITHM FOR NON-PROPORTIONAL LOADING

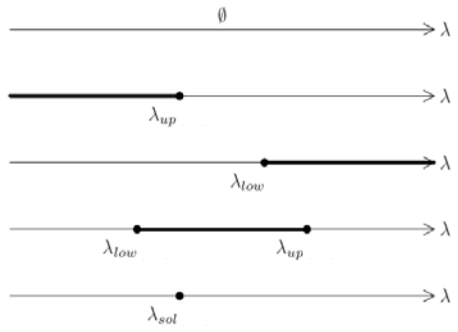
Constrained optimization:

$\max(\lambda)$ under $\sigma_{\max;i}(\lambda) \leq f_{t;i}$ for all integration points i

where $\sigma_{\max;i}(\lambda) = \begin{cases} \sigma_{1,2}(\lambda) & \text{for un-cracked integration points} \\ \sigma_{nn}(\lambda) \text{ or } \sigma_{tt}(\lambda) & \text{for cracked integration points} \end{cases}$

ADAPTED ALGORITHM FOR NON-PROPORTIONAL LOADING

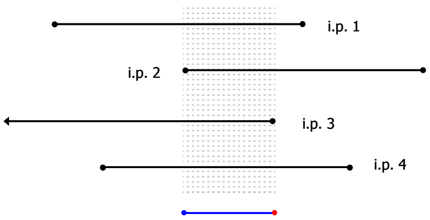
Possible solution set per integration point



ADAPTED ALGORITHM FOR NON-PROPORTIONAL LOADING

Overall solution set:

- To be determined from the individual solution sets as the intersection of these sets.
- If the overall solution set is *non-empty* take the upper bound as critical load multiplier.



ANALYSIS OF AN UN-REINFORCED MASONRY FAÇADE

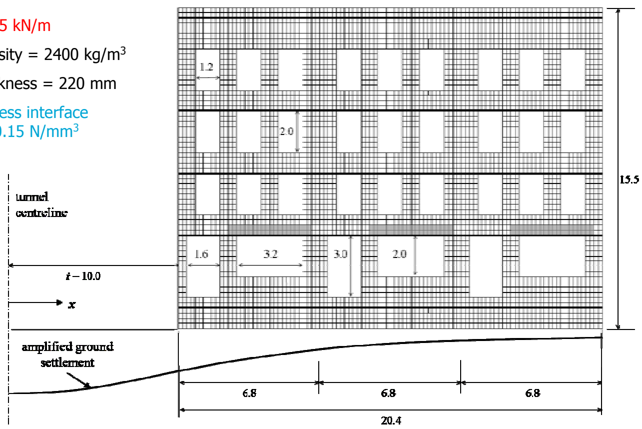
Façade geometry, mesh and loads by DeJong, Hendriks & Rots (2008)

Floor load = 5 kN/m

Masonry density = 2400 kg/m^3

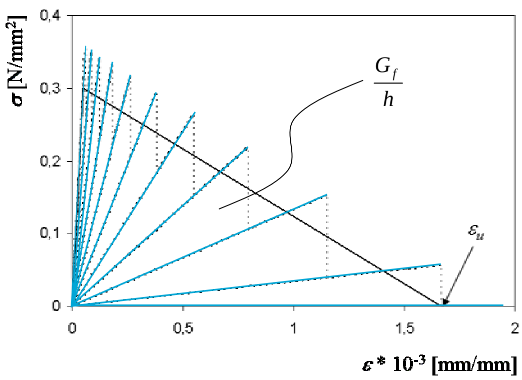
Masonry thickness = 220 mm

Normal stiffness interface elements = 0.15 N/mm^3



ANALYSIS OF AN UN-REINFORCED MASONRY FAÇADE

Applied stress-strain law for masonry



$E = 6.0$ GPa

$\nu = 0.2$

$f_t = 0.3$ N/mm^2

$G_f = 0.05$ N/mm

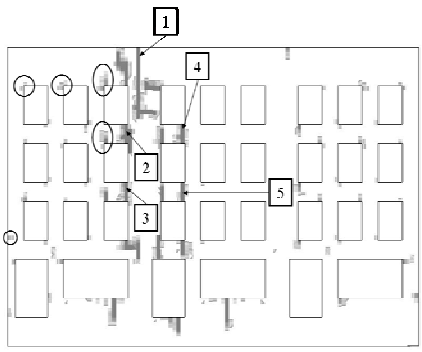
$h = 200$ mm

$\varepsilon_u = 1.667 * 10^{-3}$

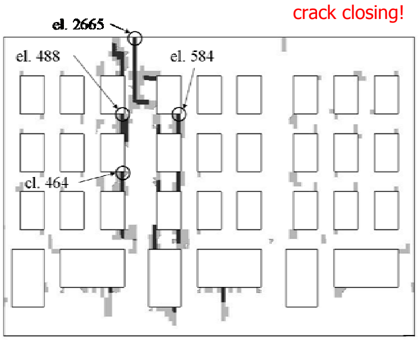
ANALYSIS OF AN UN-REINFORCED MASONRY FAÇADE

Results at step 6000

Note local crack closing!

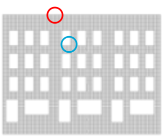


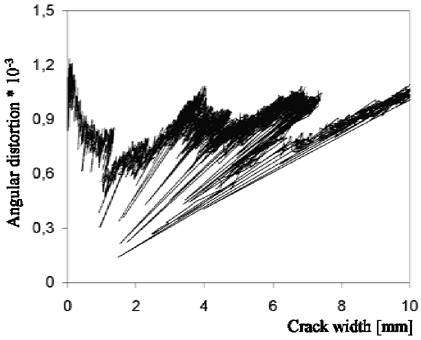
Reduced Young's moduli



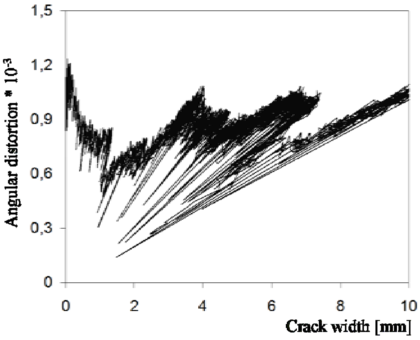
Crack strains

ANALYSIS OF AN UN-REINFORCED MASONRY FAÇADE



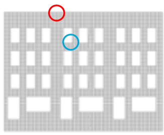


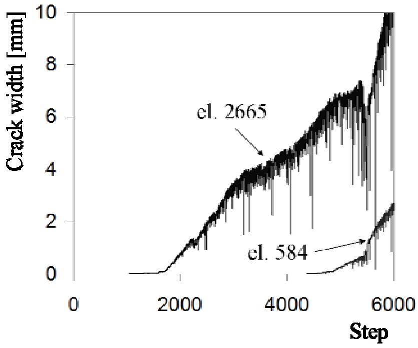
Angular distortion vs. crack width
for element 2665



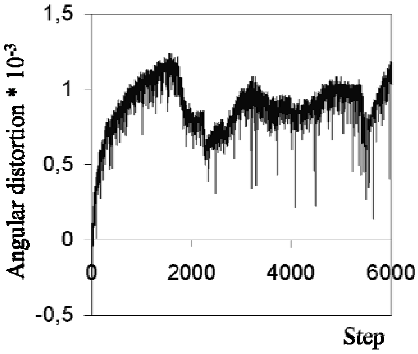
Angular distortion vs. crack width
for element 584

ANALYSIS OF AN UN-REINFORCED MASONRY FAÇADE





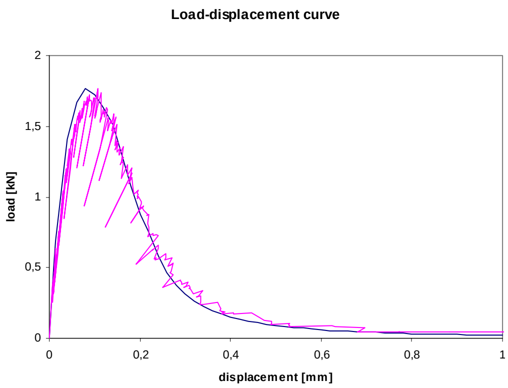
Development of crack width in elements 2665 and 584 in 'time'



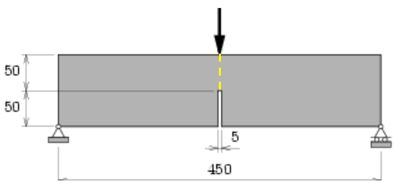
Development of angular distortion in 'time'

Current research: implementation of interface elements

Example applications: discrete cracking, bond-slip behaviour



Example taken from DIANA User's Manual – Concrete and Masonry Analysis



CONCLUSIONS

- Fracture in brittle un-reinforced structures may be modelled effectively by adopting a Sequentially Linear Analysis scheme as convergence is no longer an issue.
- Non-proportional loading conditions may be applied. However, the algorithm is more complex and new difficulties may arise.
- The implementation of interface elements opens the door to new applications: discrete cracking, bond-slip behaviour, ...

FUTURE WORK

Future research may include the following topics:

- More thorough investigation of non-proportional loading as some questions still remain unanswered.
- Implementation of a Coulomb friction model.
- Increase performance by trying to reduce the number of decompositions needed to solve the system of equations → only a few coefficients in the system stiffness matrix change due to a local stiffness reduction.

Questions

are guaranteed in life;

Answers

aren't.