

## Design of prestressed concrete structures subject to severe thermal loading

*S.J.H. Meijers, J.J. van Sloten,  
J.H.A. Strik, J.G. Kraus*



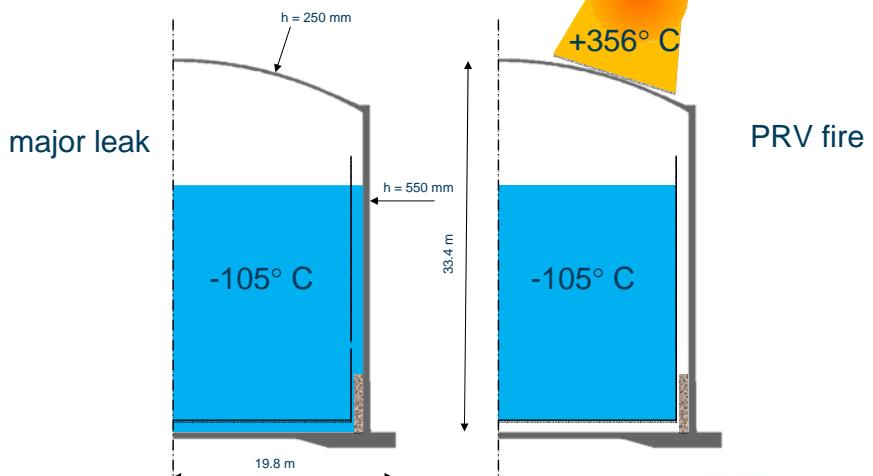
### Presentation outline

- Introduction
- Considered load cases
- Modelling
- Computation strategies for combined mechanical and thermal loading

Non-linear elastic concrete cross-section analysis  
Superposition of thermal restraint

- Comparison of results
- Conclusion
- Discussion

## Considered load cases



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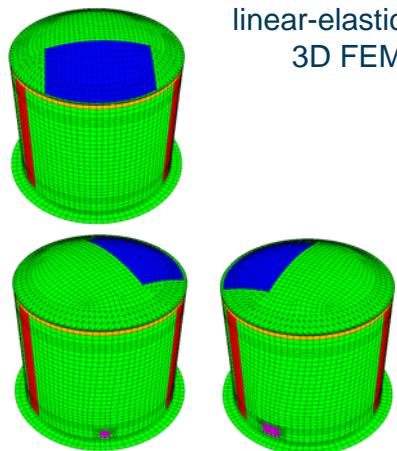
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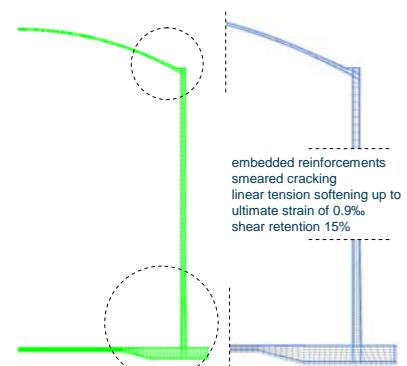
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## Modelling

linear-elastic  
3D FEM



non-linear elastic  
axi-symmetrical FEM



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## Computation strategies for combined mechanical and thermal loading

Linear-elastic 3D FEM		Non-linear elastic axi-symmetrical FEM
1	mechanical loading	thermal loading
1a	M,N,V section forces	M,N,V section forces
1b	non-linear elastic cross-section analysis	linear equations
1c	mechanical strains	thermal restraint
2	superposed strains (linear)	
3	concrete and rebar stresses	



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## Computation strategies for combined mechanical and thermal loading

Linear-elastic 3D FEM		Non-linear elastic axi-symmetrical FEM
1	mechanical loading	thermal loading
1a	M,N,V section forces	M,N,V section forces
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dedicated user subroutine in DIANA

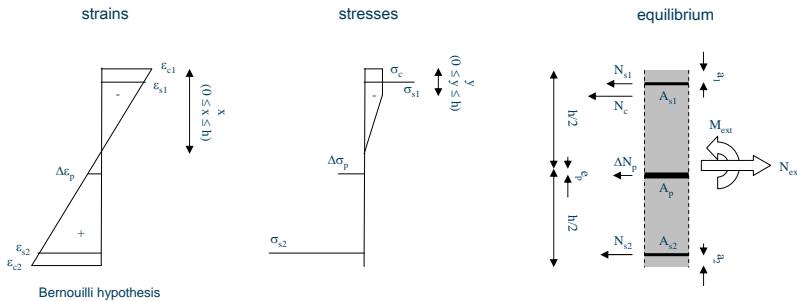
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## Non-linear elastic concrete cross-section analysis



Bernoulli hypothesis

$$\begin{cases} \epsilon_{c1} \\ \epsilon_{c2} \end{cases} \xrightarrow{\text{find roots of coupled, at least 3rd order polynomials}} \begin{cases} N_{ext} \\ M_{ext} \end{cases}$$

## Non-linear elastic concrete cross-section analysis

define

$$\mathbf{u}_j = \begin{Bmatrix} \epsilon_{c1,j} \\ \epsilon_{c2,j} \end{Bmatrix}, \quad \mathbf{r}_j = \begin{Bmatrix} N_{c,j} + N_{s1,j} + N_{s2,j} + \Delta N_{p,j} - N_i \\ M_{c,j} + M_{s1,j} + M_{s2,j} + \Delta M_{p,j} - M_i \end{Bmatrix}$$

iterate

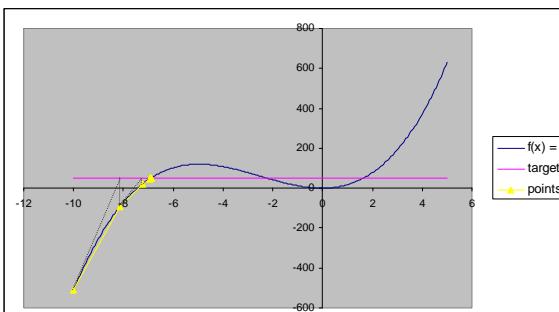
$$\mathbf{S}_j = \left( \frac{\partial \mathbf{r}}{\partial \mathbf{u}} \right)_j = \begin{bmatrix} \left( \frac{\partial r_1}{\partial u_1} \right)_j & \left( \frac{\partial r_1}{\partial u_2} \right)_j \\ \left( \frac{\partial r_2}{\partial u_1} \right)_j & \left( \frac{\partial r_2}{\partial u_2} \right)_j \end{bmatrix}$$

$$d\mathbf{u}_j = -\mathbf{S}_j^{-1} \mathbf{r}_j$$

$$\mathbf{u}_{j+1} = \mathbf{u}_j + d\mathbf{u}_j$$

until

$$\frac{|\mathbf{r}|_j}{|\mathbf{r}|_0} < Tol$$



## Superposition of thermal restraint

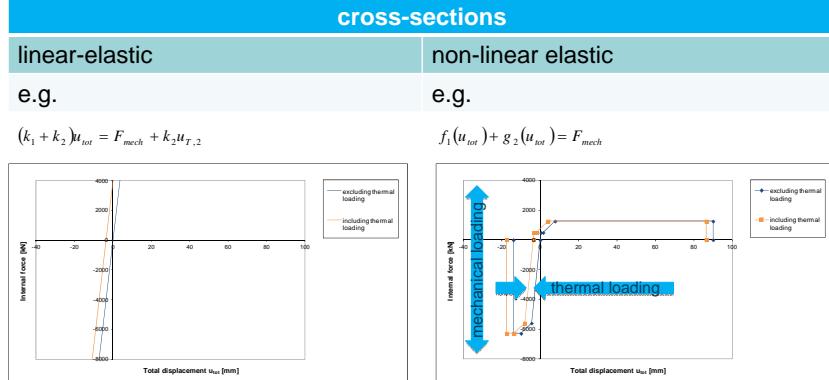
total strain as influenced by boundary conditions ↔ amount of restraint

$$\varepsilon_{tot} = \varepsilon_S + \varepsilon_T$$

strain causing stress

superimpose on strains due to mechanical loading

thermal loading in terms of strain



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## Computation strategies for combined mechanical and thermal loading revisited

Linear-elastic 3D FEM		Non-linear elastic axi-symmetrical FEM
1	mechanical loading	thermal loading
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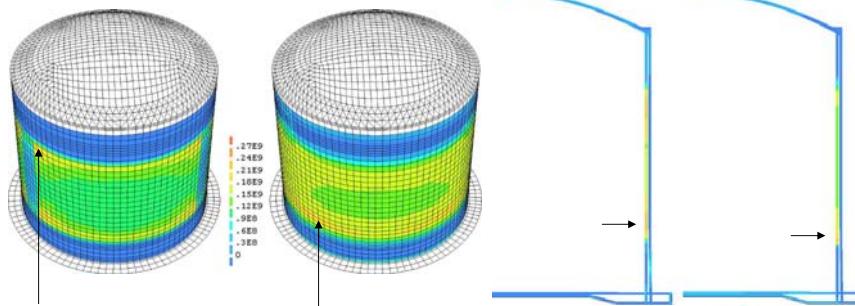
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## Comparison of results

Major leak rebar stresses			
linear-elastic 3D FEM		non-linear elastic axi-symm. FEM	
circumferential inner rebar	vertical inner rebar	vertical rebar	circumferential rebar
243~166 MPa	213 MPa	257 MPa	215 MPa



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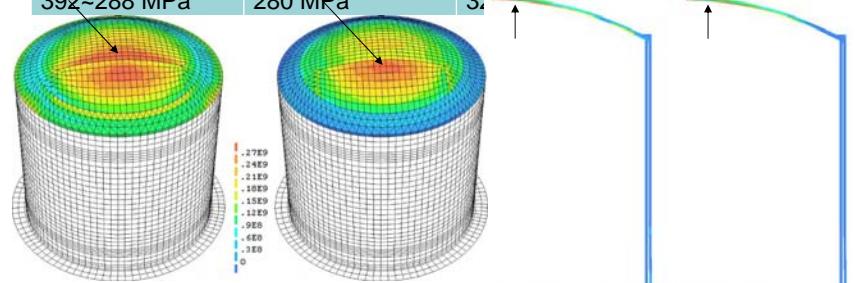
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## Comparison of results

PRV fire rebar stresses			
linear-elastic 3D FEM		non-linear elastic axi-symm. FEM	
radial/parallel inner rebar	circumferential/transv. inner rebar	radial rebar	circumferential rebar
392~288 MPa	280 MPa	320 MPa	300 MPa



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## Conclusion

- geometric discontinuities only considered in linear-elastic 3D FEM analysis
- overall fair correspondence between results of linear-elastic 3D FEM with superposition of thermal restraint and non-linear elastic axi-symmetrical FEM
- economy of thermal restraint superposition method

## Discussion