A Study on $v_{\min}$ of RC Beams without shear reinforcement

Y.Yang

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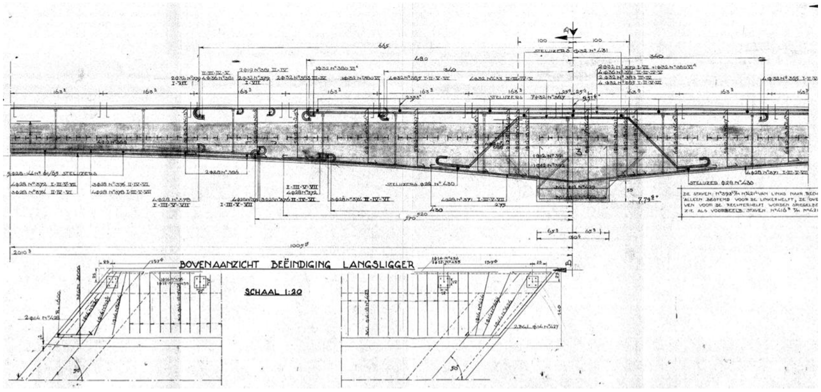
What is $v_{\min}$?

- EC shear formula:

$$V_{Rd,c} = C_{Rd,c} k(100 \rho f_{ck})^{1/3} b_w d$$

- Lower bound of $V_{Rd,c}$
- The minimum average shear stress that can cause a shear failure before the yielding of flexural reinforcement

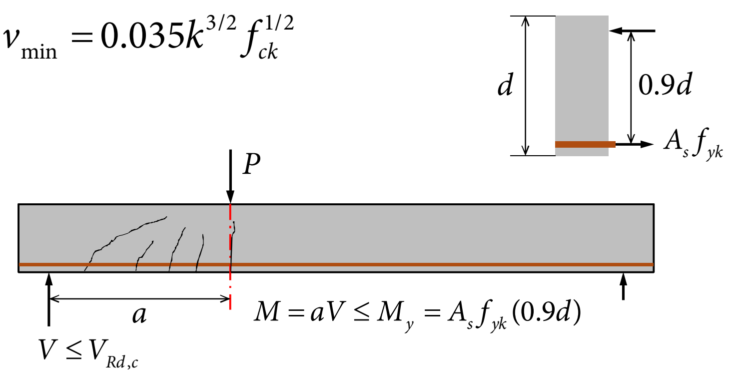
Why $v_{\min}$ is interesting for existing structures?



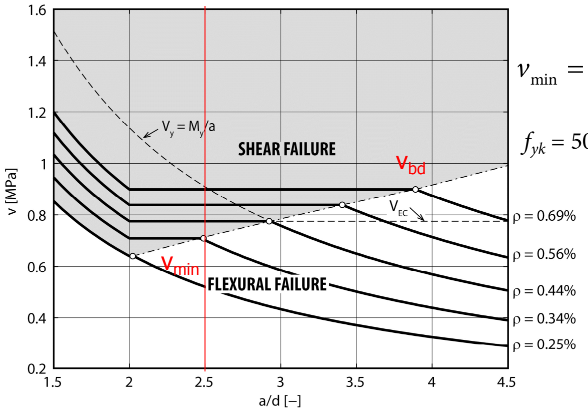
How is $v_{\min}$ determined in EC?

- In EC, $v_{\min}$ is expressed as follow:

$$v_{\min} = 0.035 k^{3/2} f_{ck}^{1/2}$$



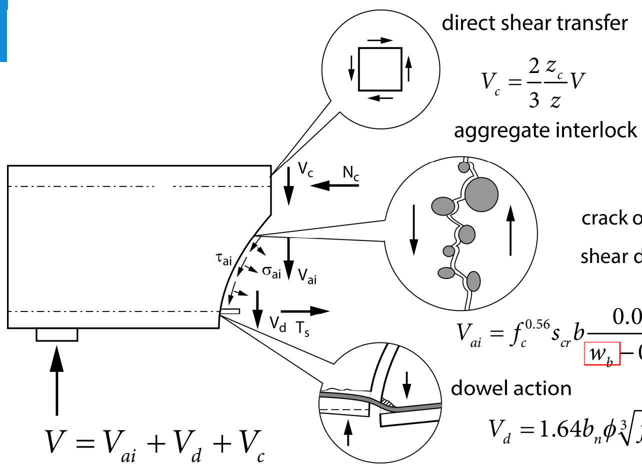
How is $v_{\min}$ determined in EC?



$$v_{\min} = 10.54\left(C_{Rd,c}k\right)^{3/2}\left(\alpha\frac{f_{ck}}{f_{yk}}\right)^{1/2}$$

f_{yk} = 500 MPa in all cases!

$v_{\min}$ based on a physical model



$$V_c = \frac{2}{3}\frac{z_c}{z}V$$

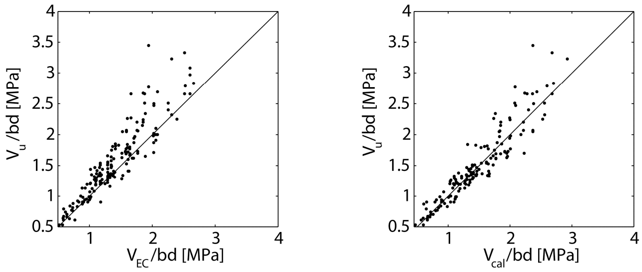
$$V_{ai} = f_c^{0.56} s_{cr} b \frac{0.03}{w_b - 0.01}\left(-978\Delta^2 + 85\Delta - 0.27\right)$$

$$V_d = 1.64 b_n \phi \sqrt[3]{f_c}$$

$$V = V_{ai} + V_d + V_c$$

$v_{\min}$ based on a physical model

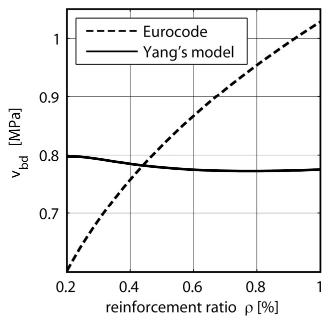
- COV_{EC} = 13.1%, COV_{cal} = 12.2%



$v_{\min}$ based on a physical model

- At v_{bd}, $\varepsilon_s = f_y/E_s$ by definition
- The crack width is predefined: $w_b = \frac{f_y}{E_s} l_{cr,m}$

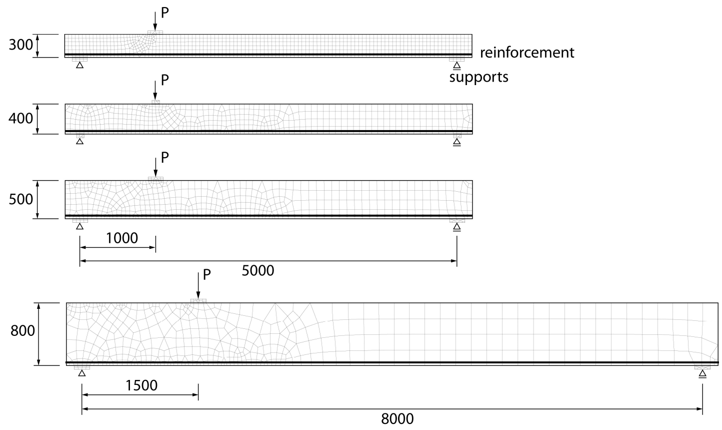
$$V_{ai} = f_c^{0.56} s_{cr} b \frac{0.03}{w_b - 0.01}(-978\Delta^2 + 85\Delta - 0.27)$$



Study with FEM – variables

- Beam depth d, d = 255, 355, 455 or 755 mm
- Concrete strength f_{cm}, f_{cm} = 34 or 68 MPa
- Reinforcement ratio ρ
- Shear slenderness ratio a/d

FEM – meshes



FEM – model configurations

- Concrete properties based on FEM guideline and Model Code 1990

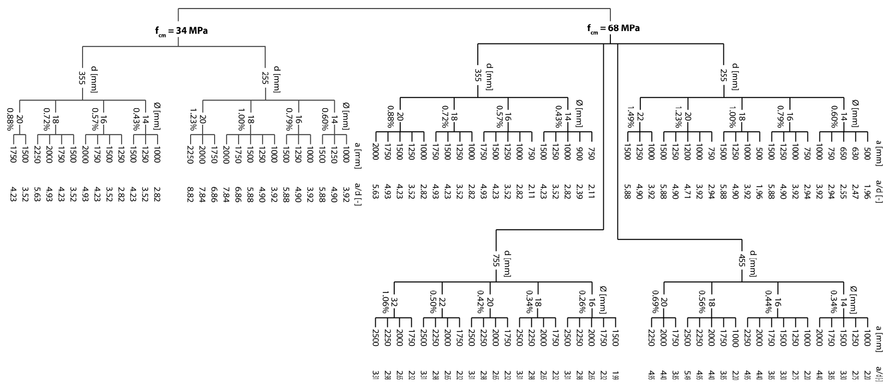
Low strength concrete	
Compressive strength f_{cm}	= 34.0 MPa
Tensile strength f_{ctm}	= 2.81 MPa
Elastic Modules E_c	= 34.0 GPa
Fracture Energy G_f	= 70.2 N/m

High strength concrete	
Compressive strength f_{cm}	= 68.0 MPa
Tensile strength f_{ctm}	= 4.46 MPa
Elastic Modules E_c	= 42.6 GPa
Fracture Energy G_f	= 111.4 N/m

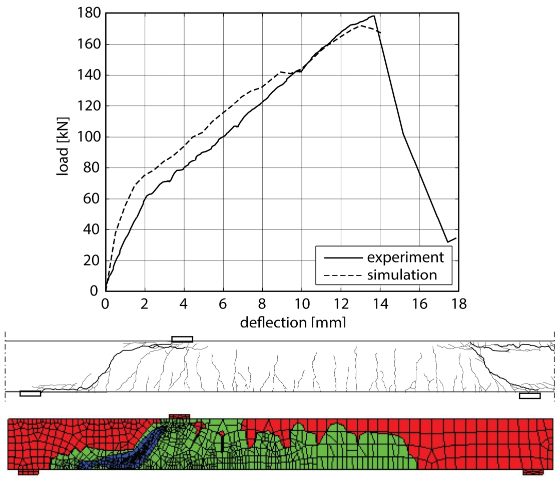
- Smeared crack model, fixed crack model when $\sigma/f_{ctm} < 0.8$
- Shear retention factor $K_n/K_s = 20$, with $\tau_u = \frac{1}{6}\frac{2}{1+120\frac{w}{16+D_{max}}}\sqrt{f_{cm}}$

FEM – models

- 115 models



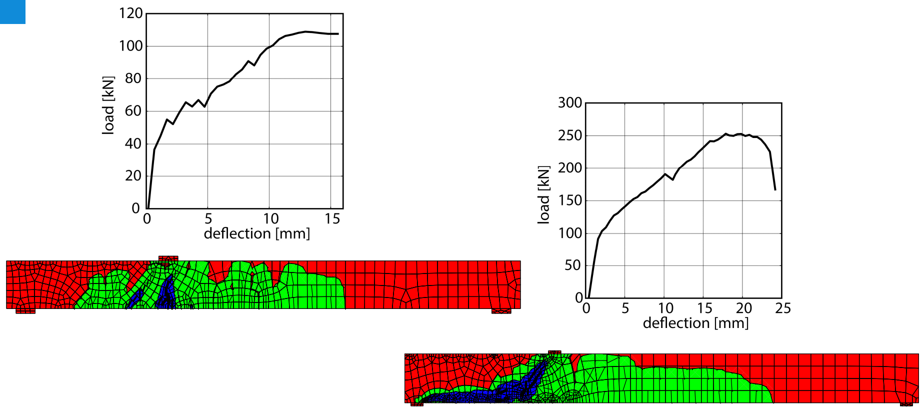
FEM – validation



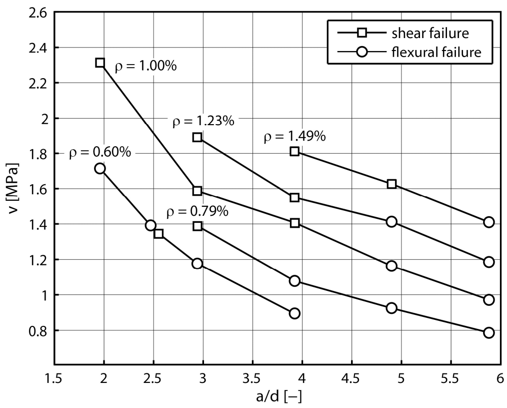
Typical failure modes

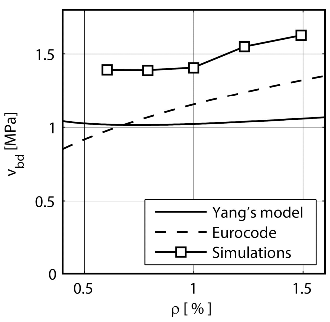
- Shear failure – opening of a critical inclined crack
- Flexural failure – yielding of longitudinal reinforcement
- Mixed failure – yielding of reinforcement first than shear failure

Typical failure modes

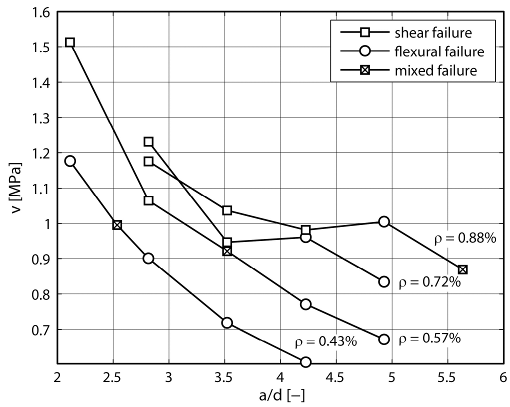


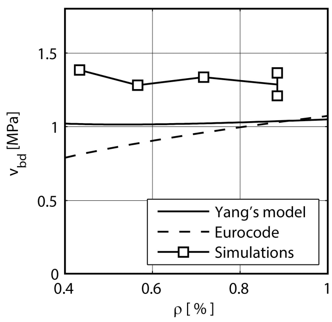
Results – d = 255 mm, f_{cm} = 68 MPa



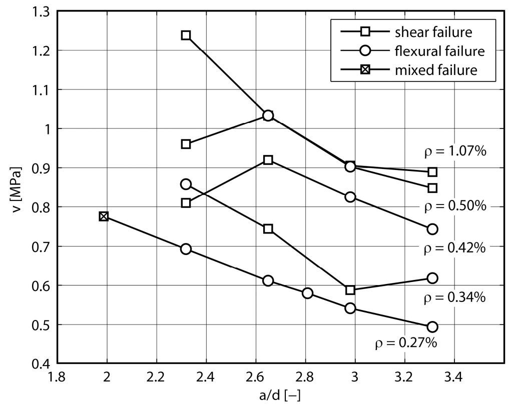


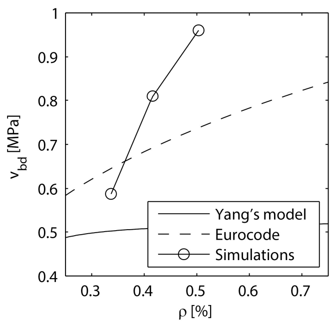
Results – d = 355 mm, f_{cm} = 68 MPa





Results – d = 755 mm, f_{cm} = 68 MPa

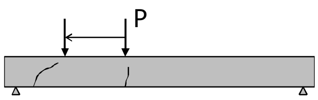




Conclusions

- FEM models reflects the influence of a/d to shear capacity
- Both the FEM models and the theoretical models implies the value of v_{bd} is stable with regard to the reinforcement ratio and a/d
- Both models suggest that a higher $v_{\min}$ than current EC formula should be taken
- FEM gives systematically larger prediction

Future experimental study



Series 1

f_{cm} [MPa]	h [mm]	rebar config.	ρ [%]	M_y [kN·m]	v_{EC} [MPa]
70	300	1 Ø10 + 2 Ø16	0.60%	69.4	0.97
70	300	3 Ø16	0.75%	87.0	1.05
70	300	1 Ø12 + 2 Ø20	0.93%	106.0	1.13
70	300	3 Ø20	1.18%	134.4	1.22

Series 2

f_{cm} [MPa]	h [mm]	rebar config.	ρ [%]	M_y [kN·m]	v_{EC} [MPa]
40	300	3 Ø12	0.44%	44.5	0.73
40	300	3 Ø16	0.75%	87.0	0.87
40	300	1 Ø12 + 2 Ø20	0.93%	106.0	0.94
70	500	2 Ø16 + 1 Ø20	0.49%	167.9	0.81
70	500	3 Ø20	0.64%	237.1	0.90

Thank you for your attention!